Benchmarking XAL with COSY and IMPACT

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XAL Workshop

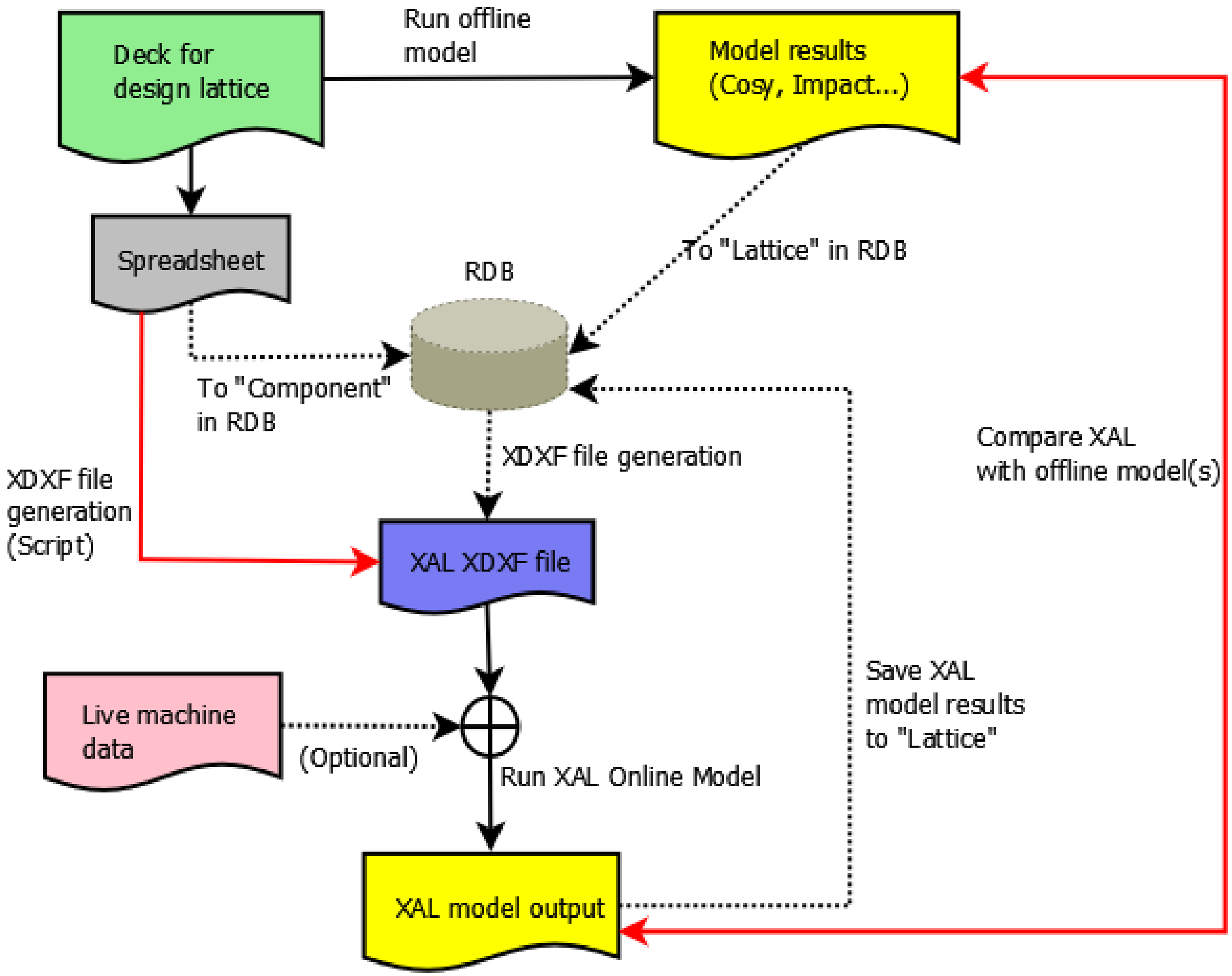
12/13/2012

Outline

- Model Benchmarking Process
- Benchmarked Elements and Examples
- Cavity Benchmarking Experience
- Some Ongoing Questions

Model Benchmarking Process

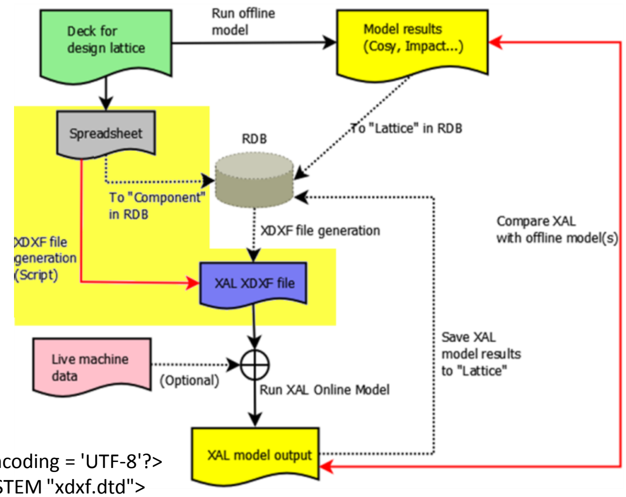
- Benchmark XAL
  - COSY
  - IMPACT
- Compare:
  - Energy
  - 6x6 Transfer Matrix
  - Phase Space (xx', yy')
  - Twiss Parameters (αβε)
  - Phase Advance
  - Dispersion
  - Chromaticity



ReA.xdxf, main.xal and model.params Files

CSV file created from Excel Lattice Spreadsheet File, or Impact file
XDXF file created from CSV file format, two versions currently

System	Subsys.	Type	FRIB Name	NSCL Name	Length	Var. 1	Var. 2	Var. 3	Var. 4	FRIB Pos.	XAL Pos.
REA	BTS3	QVE	REA_BTS3:QVE_D0954	LB004TA	0.1	-3.83279	0.03			95.37137	0.2154
REA	BTS3	QHE	REA_BTS3:QHE_D0955	LB004TB	0.1	5.511346	0.03			95.4924	0.336439
REA	BTS3	QVE	REA_BTS3:QVE_D0956	LB004TC	0.1	-3.40761	0.03			95.61344	0.457477



ReA.xdxf

```
<node id="REA_BTS3:QVE_D0954" len="0.1" pid="LB004TA" pos="0.2154" s="95.37136608" type="QVE">
      <attributes>
        <magnet dfltMagFld="-3.8327921300" len="0.1" polarity="-1"/> <aperture x="0.03"/>
      </attributes>
      <channelsuite name="electrostaticsuite">
        <channel handle="voltageRead" settable="false“ signal="LB004TA"/>
        <channel handle="voltageSetH" settable="true" signal="LB004TAHR"/>
        <channel handle="voltageSetV" settable="true" signal="LB004TAVT"/>
      </channelsuite>
    </node>
    <node id="REA_BTS3:QHE_D0955" len="0.1" pid="LB004TB" pos="0.3364385" s="95.49240458" type="QHE">
      <attributes>
        <magnet dfltMagFld="5.5113455500" len="0.1" polarity="1"/> <aperture x="0.03"/>
      </attributes>
     <channelsuite name="electrostaticsuite">
        <channel handle="voltageRead" settable="false" signal="LB004TB"/>
        <channel handle="voltageSetH" settable="true" signal="LB004TBH"/>
        <channel handle="voltageSetV" settable="true" signal="LB004TBV"/>
      </channelsuite>
    </node>
    <node id="REA_BTS3:QVE_D0956" len="0.1" pid="LB004TC" pos="0.457477" s="95.61344308" type="QVE">
      <attributes>
        <magnet dfltMagFld="-3.4076076400" len="0.1" polarity="-1"/> <aperture x="0.03"/>
      </attributes>
<channelsuite name="electrostaticsuite">
        <channel handle="voltageRead" settable="false" signal="LB004TC"/>
        <channel handle="voltageSetH" settable="true" signal="LB004TCHR"/>
        <channel handle="voltageSetV" settable="true" signal="LB004TCVT"/>
      </channelsuite>
    </node>
```

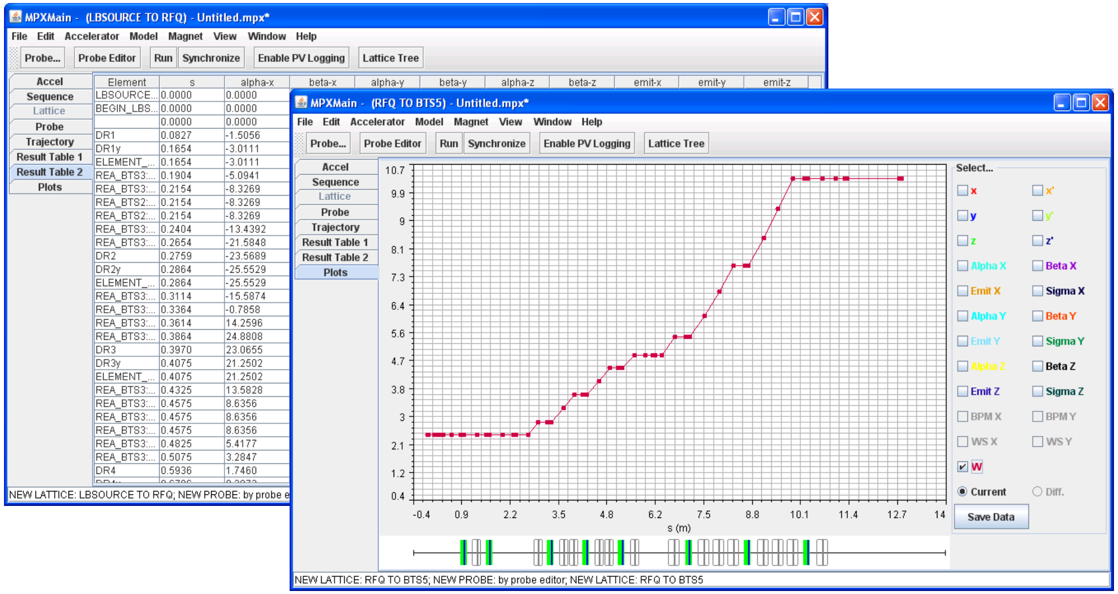
main.xal

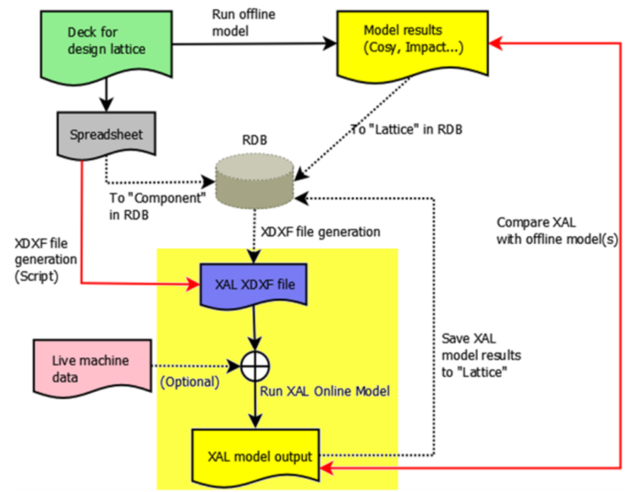
```
<?xml version = '1.0' encoding = 'UTF-8'?>
<!DOCTYPE sources SYSTEM "xdxf.dtd">
<sources>
 <deviceMapping_source name="deviceMapping" url="frib.impl"/>
 <optics_source name="optics" url="REA.xdxf"/>
 <timing_source name="timing" url="timing_pvs.tim"/>
 <tablegroup_source name="modelparams" url="REAmodel.params"/>
</sources>
```

model.params

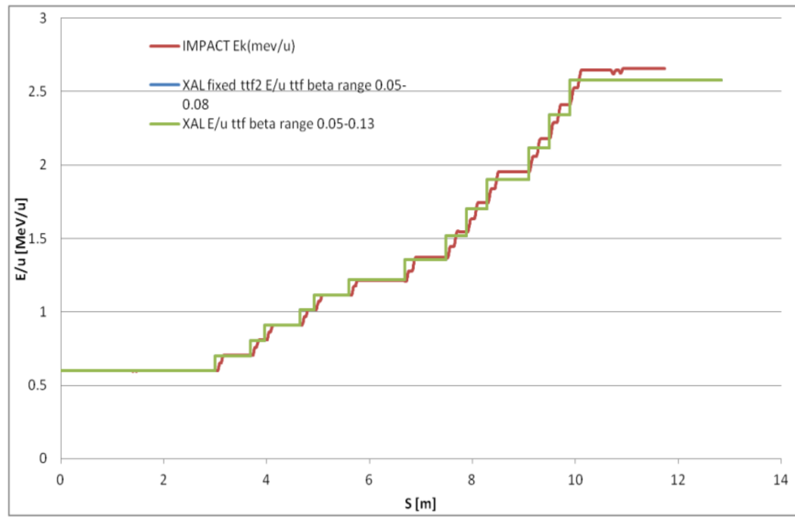
```
<table name="species">
<record name="HELIUM" mass="3.72597728E9" charge="1"/>
</table> …
<table name="twiss">
<record name="LBSOURCE TO RFQ" coordinate="x" alpha="0"
beta=".5492991115980524E-01" emittance=".1884000000000000E-03"/>
   <record name="LBSOURCE TO RFQ" coordinate="y" alpha="0"
beta=".5492991115980524E-01" emittance=".1884000000000000E-03"/>
   <record name="LBSOURCE TO RFQ" coordinate="z" alpha="0"
beta=".9609400000000001E-01" emittance=".1040647699127937E-10"/>
</table> …
<table name="location">
<record name="LBSOURCE TO RFQ" species="HELIUM" W="4.8047E4" s="0.0"/>
</table>
```

Run XAL Model—MPX, Matlab






```
  ⋮
%Run the XAL Online Model
%Read the accelerator
accl = XMLDataManager.acceleratorWithPath(acceleratorpath);
seq0 = accl.getSequence(sequencename);
%Model and Probe initializations
model = Scenario.newAndImprovedScenarioFor(seq0);
initProbe = ProbeFactory.getEnvelopeProbe(seq0, EnvTrackerAdapt(seq0));
model.resetProbe();
model.setProbe(initProbe);
model.setSynchronizationMode(Scenario.SYNC_MODE_DESIGN);
%Run model
model.run();
probe = model.getProbe();
traj = probe.getTrajectory();
  ⋮
```

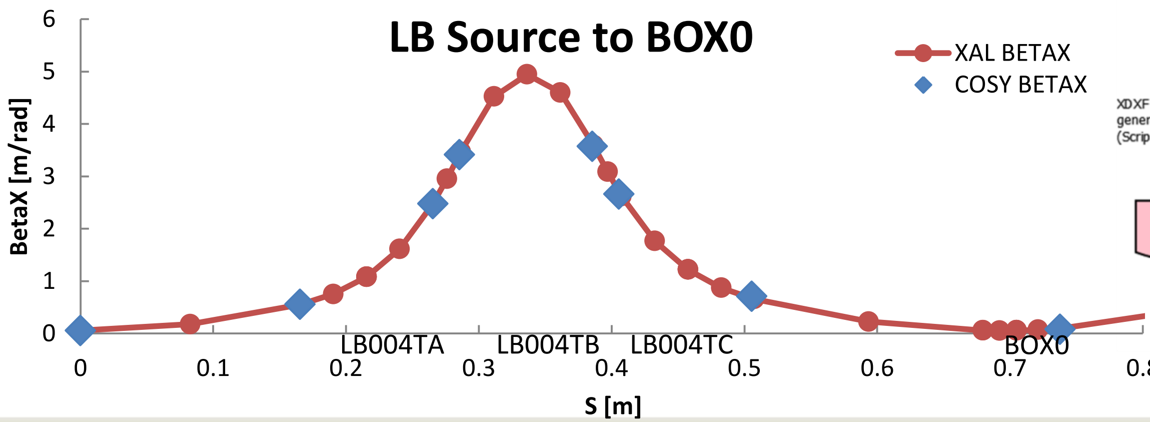


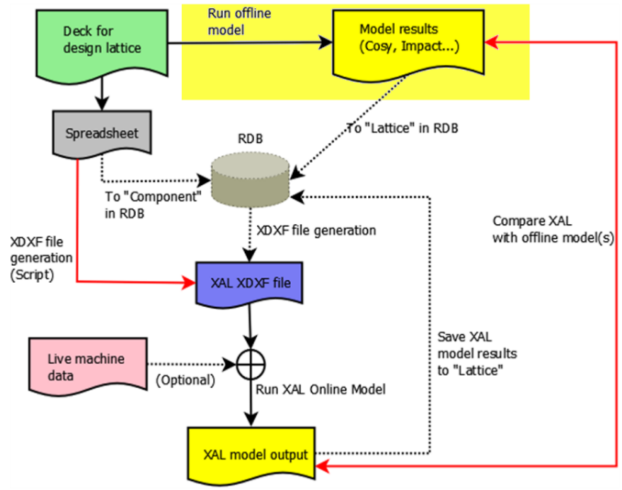
Run COSY Model, and Compare

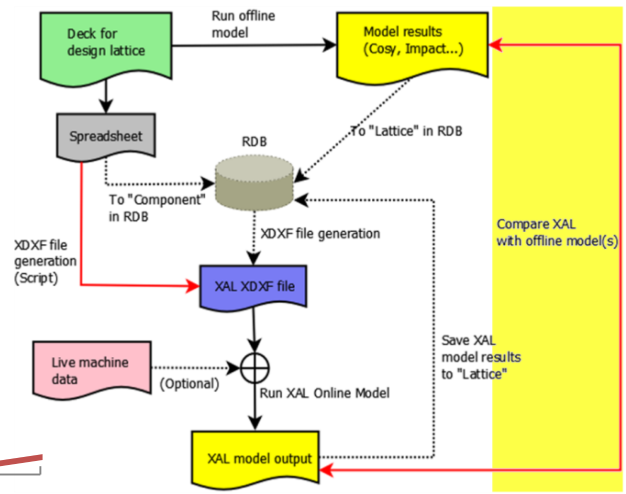
```
 PHASE SPACE 6
SPOS:   .5054000000000000

----  PHASE SPACE PARAMETERS----
 E0[MeV]=   .4804700000000000E-01
 CHIM[Tm]=   .6311737672167199E-01
 V0/CLIGHT=   .5078364094136823E-02
 A0[amu]=   4.000000000000000
 E0[MeV]=   .4804700000000000E-01
 Z0[e units]=   1.000000000000000
-- X-A                    --
 EPSXN,   .9567761329388078E-06
 EPSX,   .1883999999999998E-03
 BETAX,   .7110814434861047  ALPHAX,   3.163199329699511
 PX =  .1157E-01 PA =  .5400E-01
  R12 = -.9535E+00
-- Y-B                    --
 EPSYN,   .9567761329388087E-06
 EPSY,   .1884000000000000E-03
 BETAY,   1.478984457766634  ALPHAY,   3.452747755090183
 PY =  .1669E-01 PB =  .4057E-01
  R34 = -.9605E+00
   ⋮
```

- Create COSY ".fox" file
- Run COSY (Cosy8a)
- Matlab script converts COSY output to Excel file







Outline

- ✓ Model Benchmarking Process
- ❑ Benchmarked Elements and Examples
- ❑ Cavity Benchmarking Experience
- ❑ Some Ongoing Questions

Benchmarked Elements Summary

Elements:

- Drift
- B-Quad
- E-Quad
- Solenoid
- Dipole
- Sextupole
- Spherical Bend
- Cylindrical Bend
- Cavity
    - E0TL input (TTF=1)
    - TTF Polynomial input
        - 1Gap
        - 2Gaps

Beamlines:

- FODO Lattice
- LB Line (before RFQ)
- ReA3 Linac
- FRIB Segment1 to Stripper

Benchmark Element Example

Horiz. Dipole

COSY

Pos[m]	E[MeV]	EPSX	BETAX	ALPHAX	EPSY	BETAY	ALPHAY
0	0.048047	3	0.75	-0.7	5	1	1.3
0.471238898	0.048047	3	1.1526	0.029	5	0.372136671	0.032367

XAL

Pos[m]	E[MeV]	EPSX	BETAX	ALPHAX	EPSY	BETAY	ALPHAY
0	0.048047	3	0.75	-0.7	5	1	1.3
0.235619449	0.048047	3	1.041888506	-0.47447	5	0.536728884	0.666184
0.471238898	0.048047	3	1.1526	0.029	5	0.372136671	0.032367

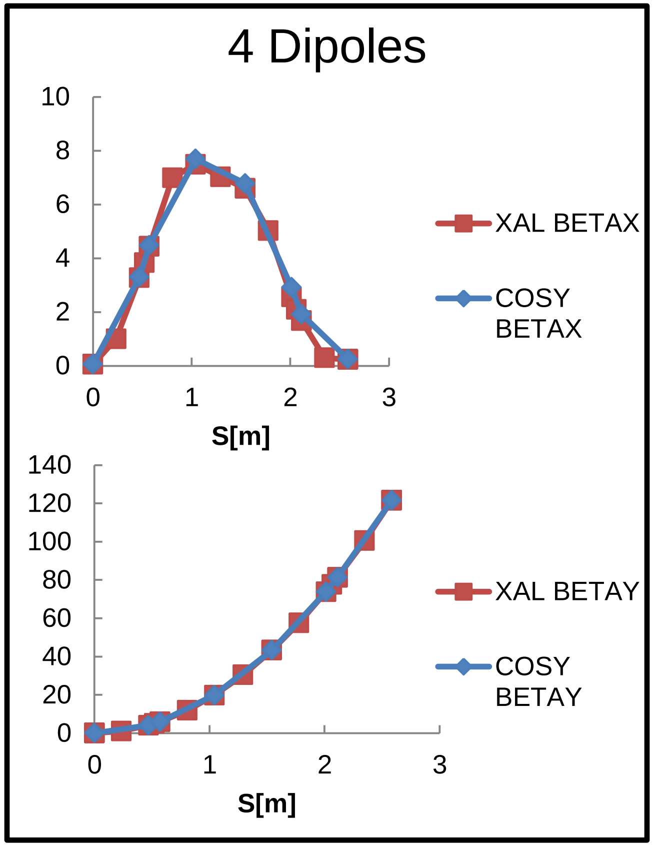
Difference

Pos[m]	E[MeV]	EPSX	BETAX	ALPHAX	EPSY	BETAY	ALPHAY
0	0	0	0	0	0	0	0
0.471239	0	0	-9.26708E-11	9.97E-11	0	2.49017E-12	1.03E-10

Percent Difference

Pos[m]	E[MeV]	EPSX	BETAX	ALPHAX	EPSY	BETAY	ALPHAY
0	0	0	-1.4803E-14	-3.2E-14	-5.3E-14	3.33067E-14	8.54E-14
0.471239	0	0	-8.04015E-09	3.44E-07	-3.6E-14	6.69156E-10	3.2E-07

Single Dipole ←



XAL

0.707106781	0.424264	0	0	0	0.175736
-1.1785113	0.707107	0	0	0	0.707107
0	0	1	0.471238898	0	0
0	0	0	1	0	0
-0.70710678	-0.17574	0	0	1	0.424252
0	0	0	0	0	1

Transform

1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1.999987	0
0	0	0	0	0	0.49999

COSY

0.7071068	0.424264	0	0	0	0.087869
-1.178511	0.707107	0	0	0	0.353556
0	0	1	0.4712389	0	0
0	0	0	1	0	0
-0.3535557	-0.08787	0	0	1	0.106064
0	0	0	0	0	1

XAL Converted to COSY Coordinates

0.707106781	0.424264	0	0	0	0.087866
-1.178511302	0.707107	0	0	0	0.353547
0	0	1	0.471239	0	0
0	0	0	1	0	0
-0.35355567	-0.08787	0	0	1	0.106062
0	0	0	0	0	1

Difference

1.88135E-08	3.13E-08	0	0	0	2.26E-06
3.02074E-07	1.88E-08	0	0	0	9.15E-06
0	0	0	2E-09	0	0
0	0	0	0	0	0
-2.986E-08	2.17E-09	0	0	0	2.69E-06
0	0	0	0	0	0

Percent Difference

2.66062E-08	7.38E-08	0	0	0	2.58E-05
-2.56318E-07	2.66E-08	0	0	0	2.59E-05
0	0	0	4.24E-09	0	0
0	0	0	0	0	0
8.44573E-08	-2.5E-08	0	0	0	2.54E-05
0	0	0	0	0	0

XAL-COSY Benchmarking Elements

Drift

Difference

Pos[m]	E[MeV]	EPSX	BETAX	ALPHAX	EPSY	BETAY	ALPHAY
0	0	0	0	0	0	0	0
0.1	0	0	0	0	0	0	0

Percent Difference

Pos[m]	E[MeV]	EPSX	BETAX	ALPHAX	EPSY	BETAY	ALPHAY
0	0	0	-1.5E-16	-3.2E-16	-5.3E-16	3.33E-16	8.54E-16
0.1	0	0	-2.4E-16	-3.7E-16	-3.6E-16	0	0

Bquad

Difference

Pos[m]	E[MeV]	EPSX	BETAX	ALPHAX	EPSY	BETAY	ALPHAY
0	0	0	0	0	0	0	0
0.1	0	0	0.000682	-0.00618	0	-0.00092	0.009894

Percent Difference

Pos[m]	E[MeV]	EPSX	BETAX	ALPHAX	EPSY	BETAY	ALPHAY
0	0	0	-1.5E-16	-3.2E-16	-5.3E-16	3.33E-16	8.54E-16
0.1	0	2.96E-16	0.000943	-0.00654	-3.6E-16	-0.00094	-0.00894

Sextupole

Difference

Pos[m]	E[MeV]	EPSX	BETAX	ALPHAX	EPSY	BETAY	ALPHAY
0	0	0	0	0	0	0	0
0.1	0	0	0	0	0	0	0

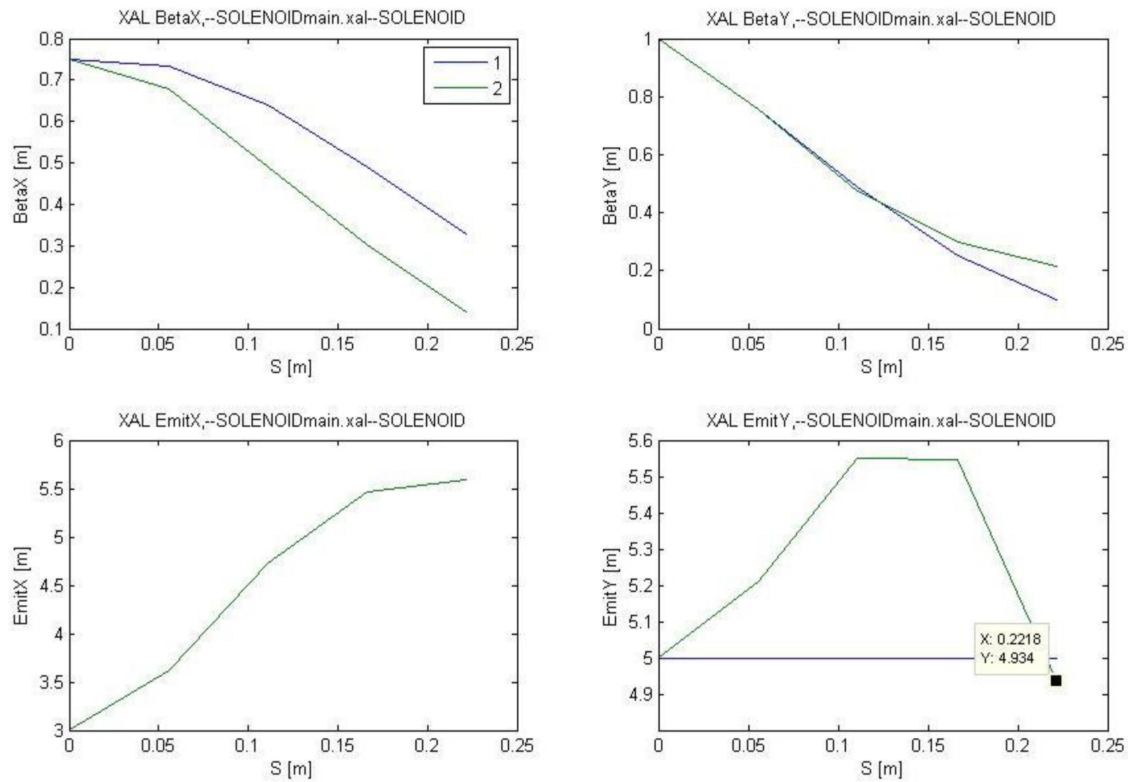
Percent Difference

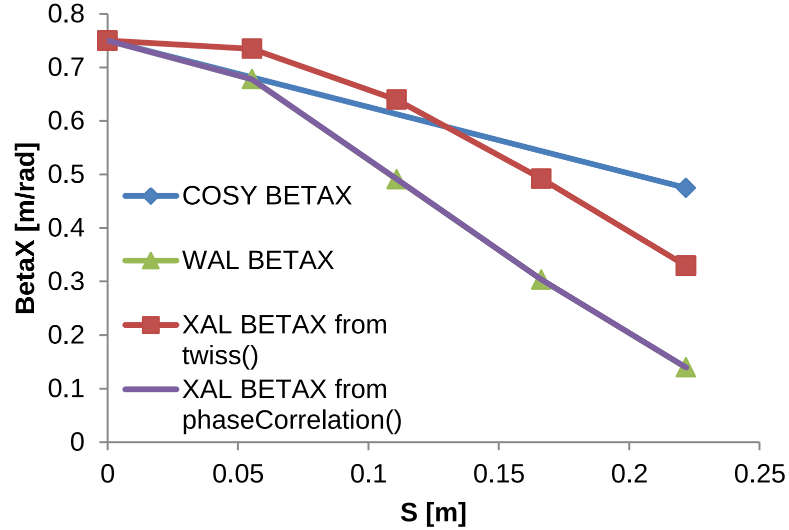
Pos[m]	E[MeV]	EPSX	BETAX	ALPHAX	EPSY	BETAY	ALPHAY
0	0	0	-1.5E-16	-3.2E-16	-5.3E-16	3.33E-16	8.54E-16
0.1	0	0	-1.5E-16	-3.2E-16	-5.3E-16	3.33E-16	8.54E-16

Benchmark:
- Energy Gain
- Phase Space (xx’,yy’)
- Twiss (Transverse)
- Rmatrix Elements

Still Need:
- Phase Advance
- Dispersion
- Chromaticity

Solenoid Only

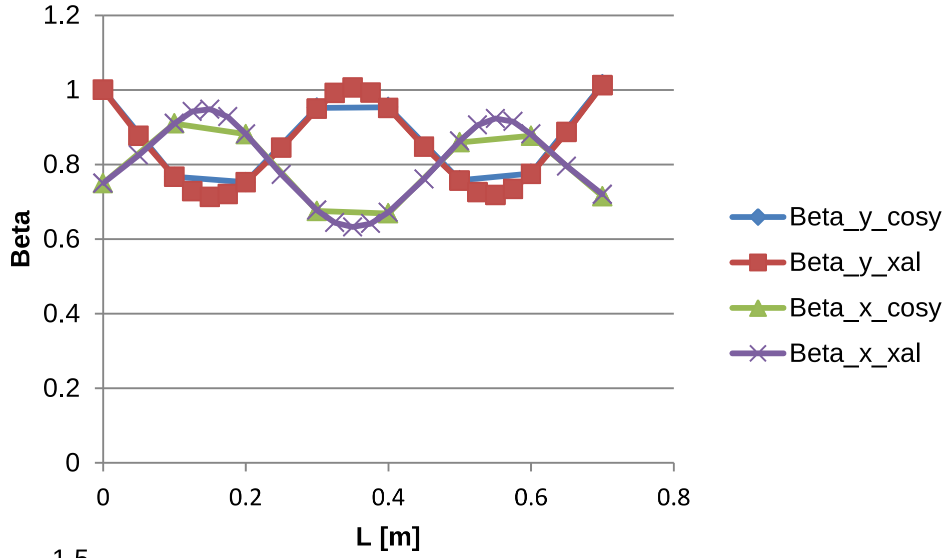


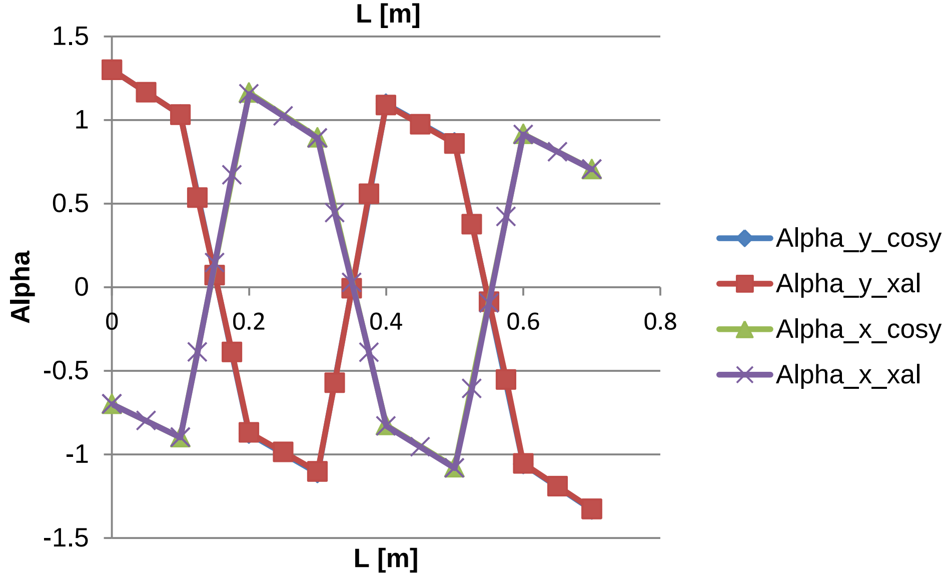


(XAL, WAL hard-edge, COSY tanh model)

Two methods for getting Twiss parameters

FODO Lattice





Rmat

COSY

0.090483	0.566283	0	0
-1.75145	0.090483	0	0
0	0	1.647233	0.748878
0	0	2.287925	1.647233

XAL

0.094178	0.566917	0	0
-1.74828	0.094178	0	0
0	0	1.645628	0.748914
0	0	2.28076	1.645628

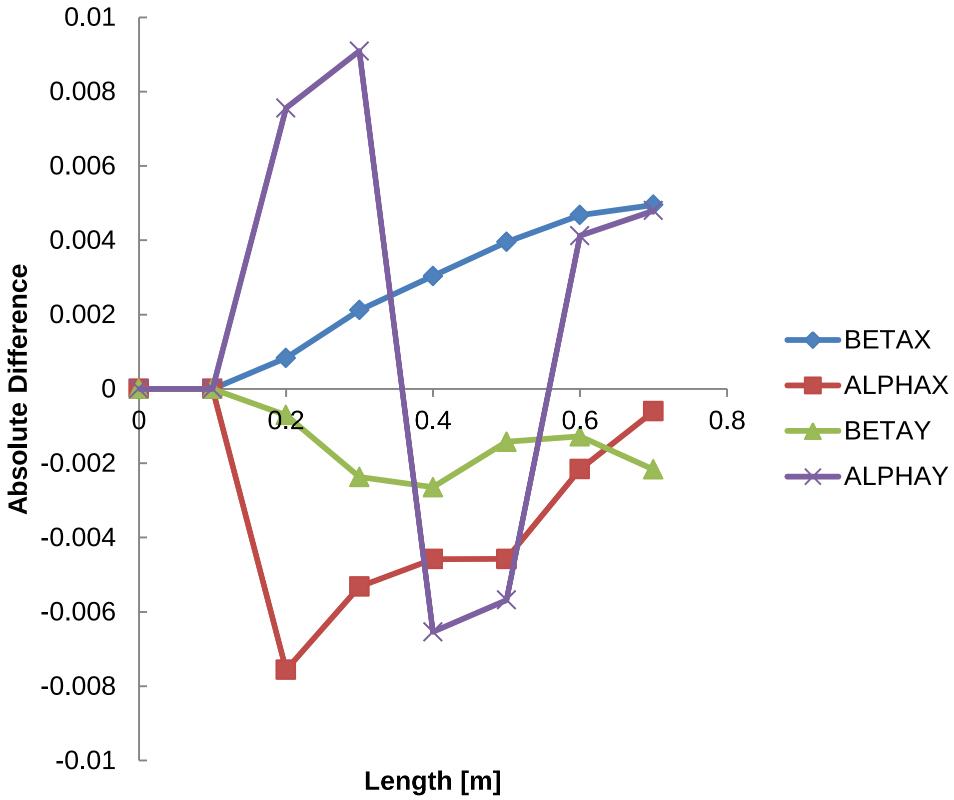
Difference

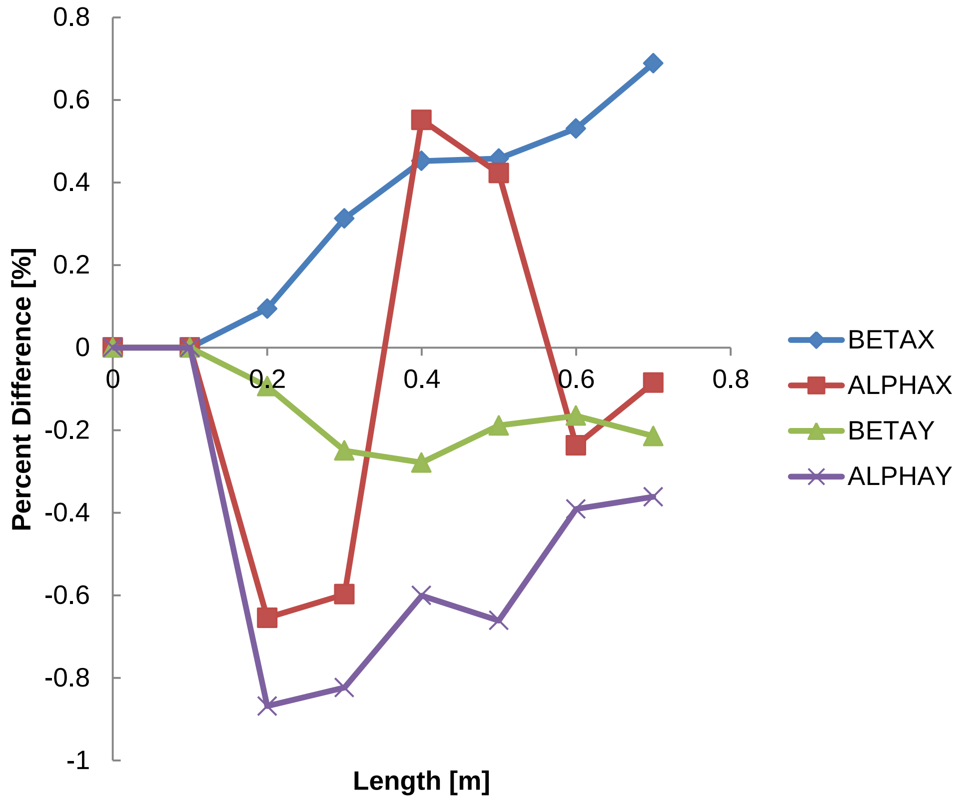
0.003695	0.000635	0	0
0.003164	0.003695	0	0
0	0	-0.0016	3.6E-05
0	0	-0.00717	-0.0016

Percent Difference

3.923672	0.111961	0	0
-0.181	3.923672	0	0
0	0	-0.09752	0.004805
0	0	-0.31416	-0.09752

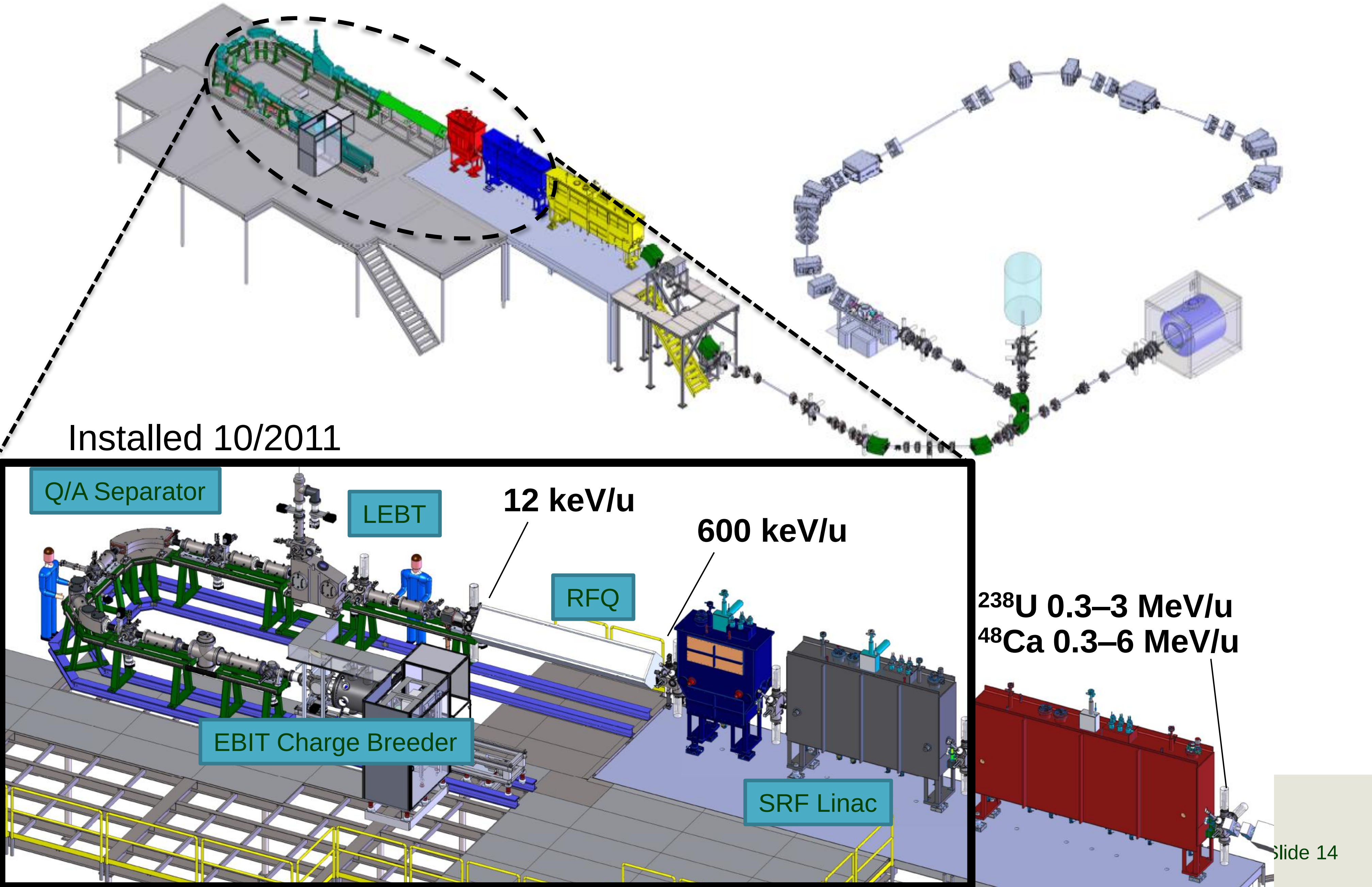
FODO Lattice XAL-COSY Percent Difference



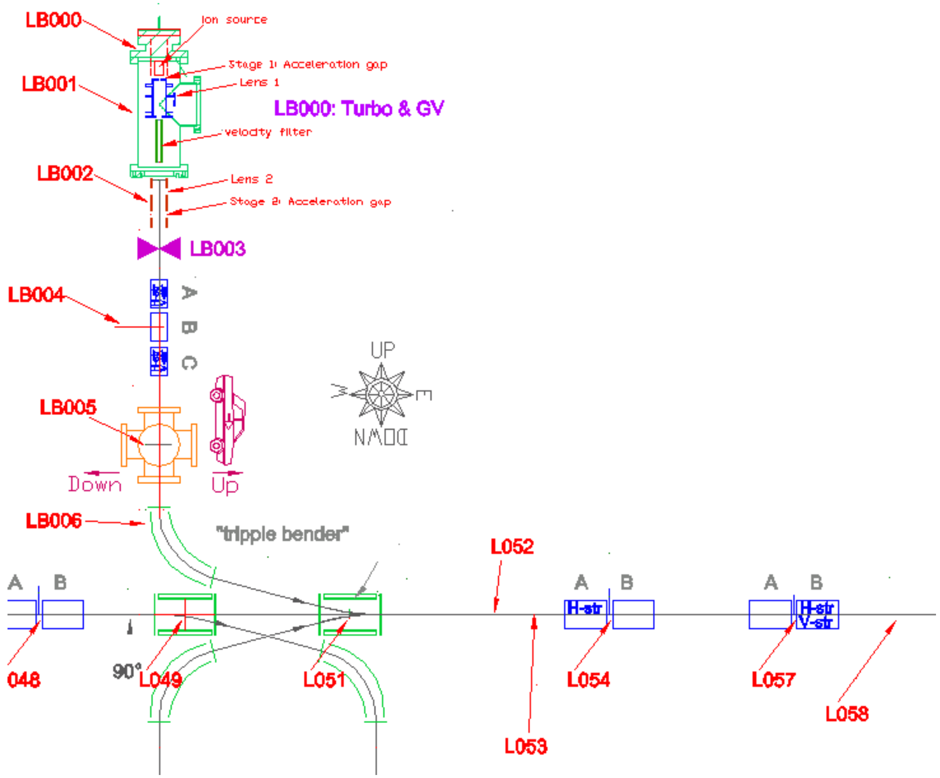


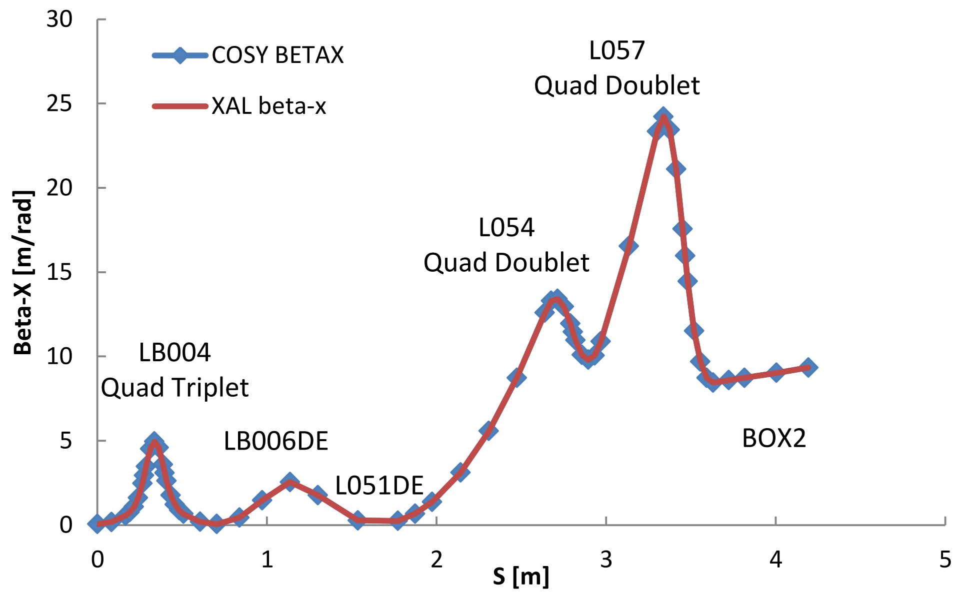
- Slight growth in the difference of the betatron function
- May be a product of code numerical output precision
- Sufficiently small for simulation benchmarking needs.

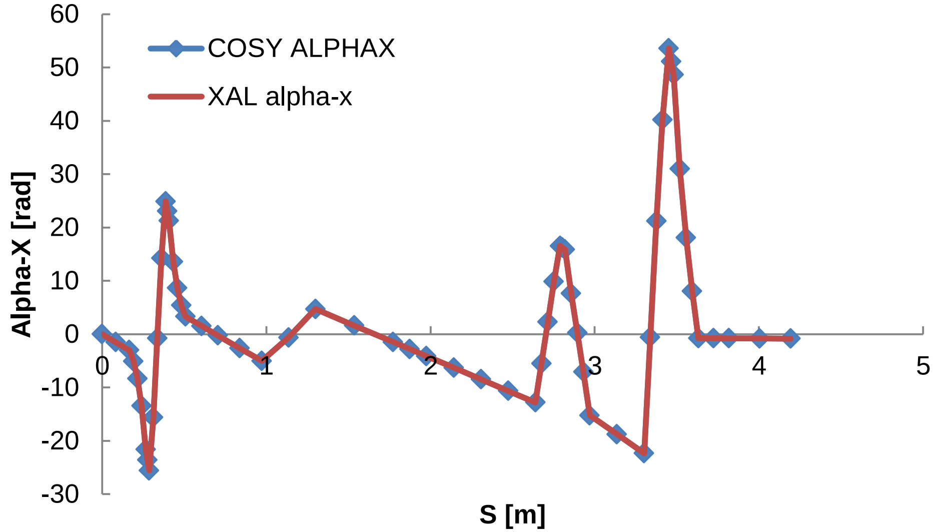
ReA3 Layout


Installed 10/2011
Q/A Separator
LEBT
12 keV/u
600 keV/u
RFQ
EBIT Charge Breeder
SRF Linac
^{238}U 0.3–3 MeV/u
^{48}Ca 0.3–6 MeV/u

Example beamline: ReA3 LB Line







- Generate xal files from impact model files
- Learn that small differences in element positions cause models to not match (errors compound)

Outline

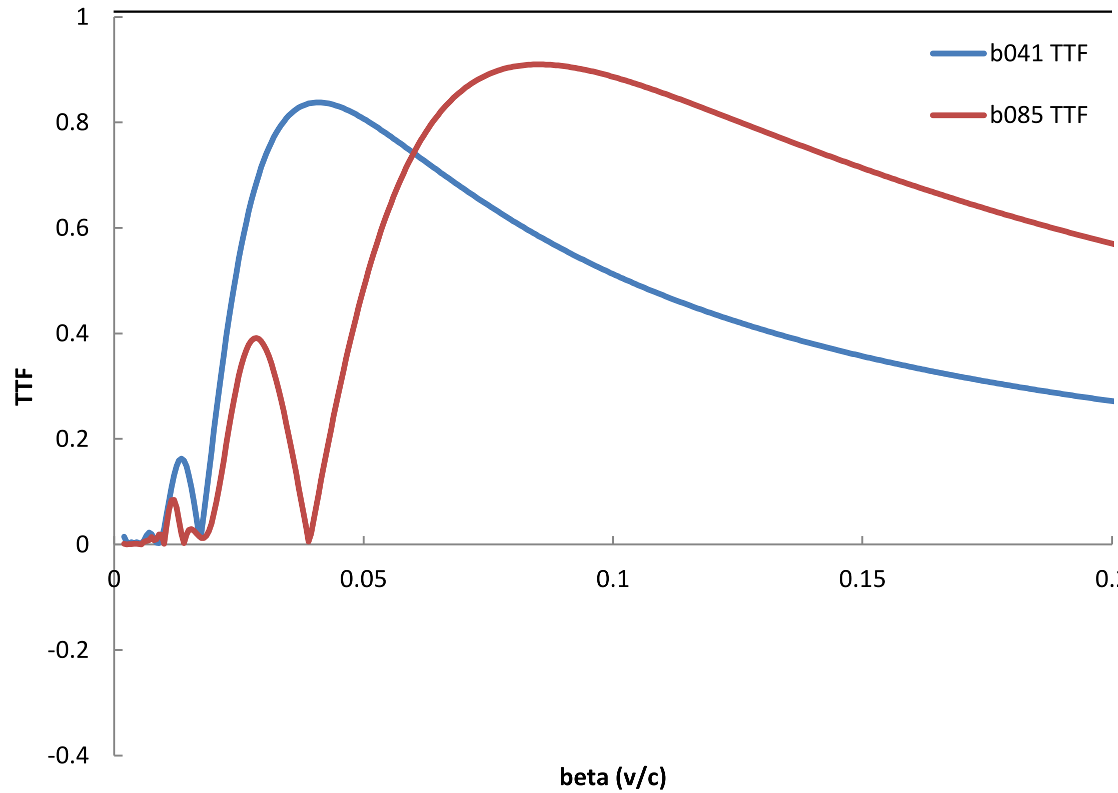
- ✓ Model Benchmarking Process
- ✓ Benchmarked Elements and Examples
- ❑ Cavity Benchmarking Experience
- ❑ Some Ongoing Questions

Energy Gain in a ¼ Wave Resonator

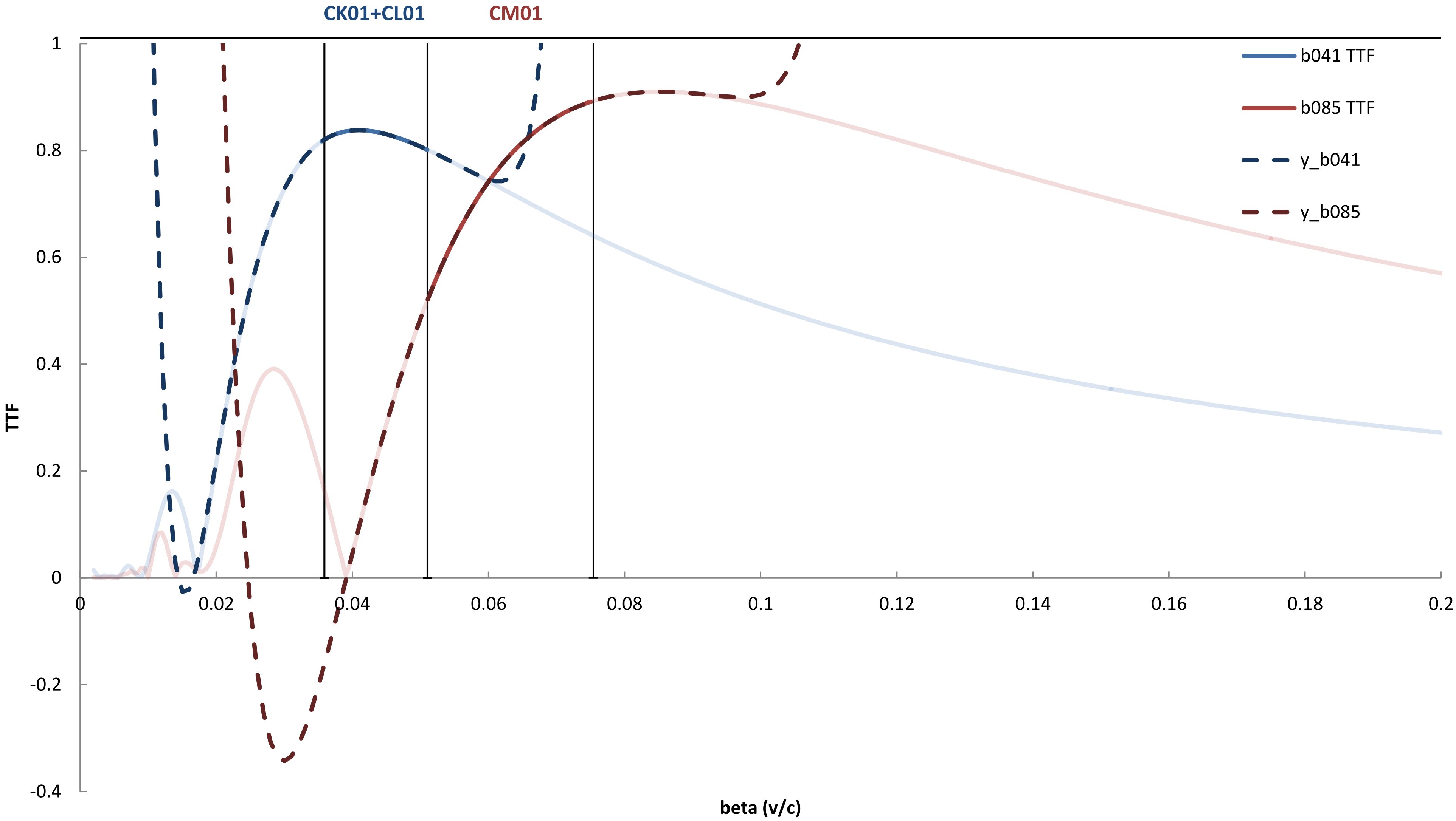
$$\Delta E = \frac{Q}{A} \times Amp \times V_0 \times TTF \times \cos\varphi$$

β = 0.041

β = 0.085



TTF Curve Polynomial Fit



Change of Basis: Longitudinal Coordinates

Units

$$\text{XAL}\begin{pmatrix} z \\ z' \end{pmatrix};\ \text{COSY}\begin{pmatrix} l \\ \frac{\Delta W}{W} \end{pmatrix};\ \text{USPAS}\begin{pmatrix} z \\ \frac{\Delta p}{p} \end{pmatrix}$$

Conversion

$$\begin{pmatrix} z \\ z' \end{pmatrix} = \begin{pmatrix} \frac{\gamma+1}{\gamma} & \frac{1}{\gamma(\gamma+1)} \\ 0 & \frac{1}{\gamma(\gamma+1)} \end{pmatrix} \begin{pmatrix} l \\ \frac{\Delta W}{W} \end{pmatrix} \qquad R_{\text{cosy}} = T^{-1} R_{\text{xal}} T$$

$$\begin{pmatrix} z \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\gamma^2} \end{pmatrix} \begin{pmatrix} z \\ \frac{\Delta p}{p} \end{pmatrix} \qquad R_{\text{uspas}} = T^{-1} R_{\text{xal}} T$$

$$\begin{pmatrix} z \\ \frac{\Delta p}{p} \end{pmatrix} = \begin{pmatrix} \frac{\gamma+1}{\gamma} & \frac{1}{\gamma(\gamma+1)} \\ 0 & \frac{\gamma}{(\gamma+1)} \end{pmatrix} \begin{pmatrix} l \\ \frac{\Delta W}{W} \end{pmatrix} \qquad R_{\text{cosy}} = T^{-1} R_{\text{uspas}} T$$

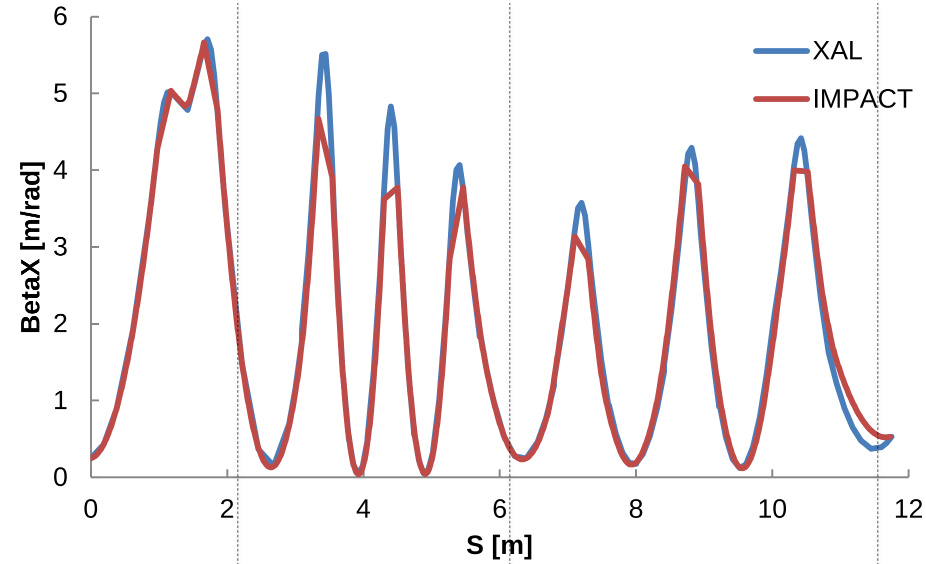
In XAL code, R56 elements:

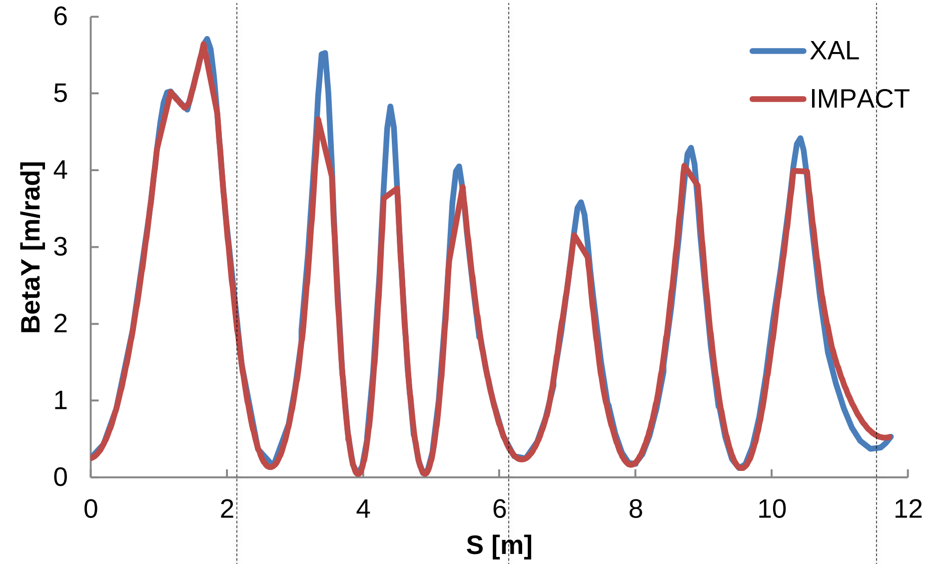
$$dz = \frac{L}{(\beta\gamma)^2} \quad \Rightarrow \quad dz = L$$

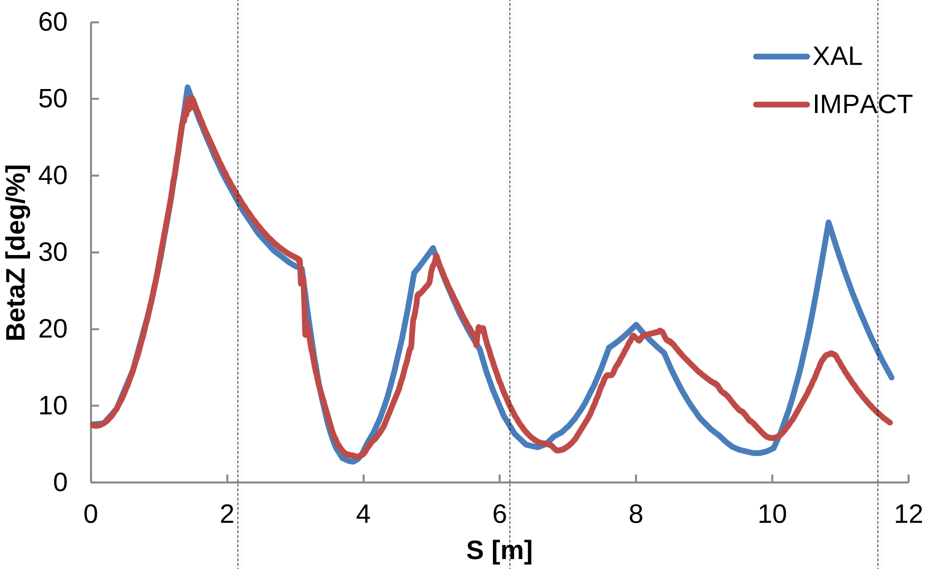
Also, for IMPACT comparison:

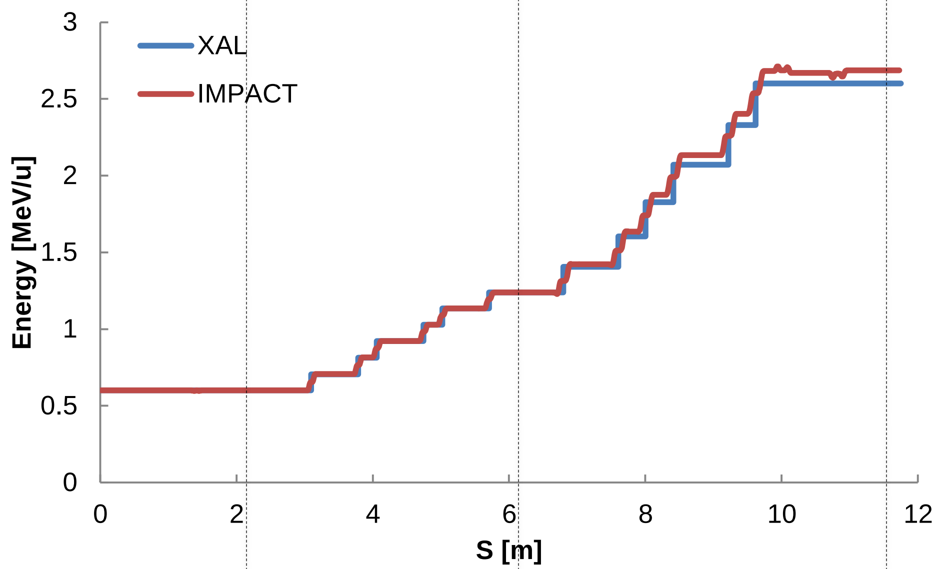
$$\beta_z[\deg/\%] = \frac{3.6 f}{\beta c \gamma(\gamma+1)} \beta[m/rad]$$

Benchmarking ReA3 Linac



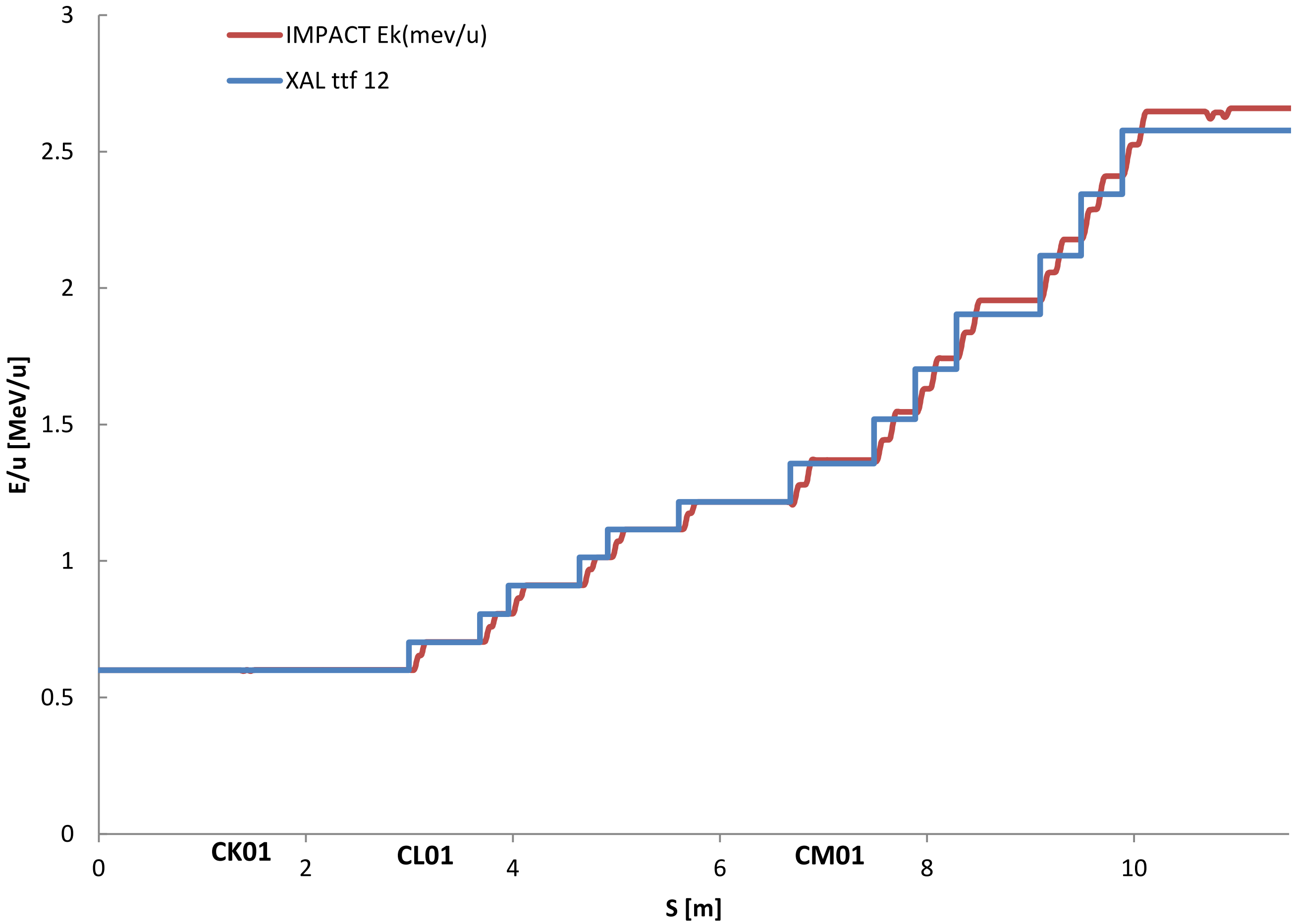






Energy Gain ReA Linac

$$\Delta E = \frac{Q}{A} \times Amp \times V_0 \times TTF \times \cos\varphi$$



<u>b041</u>

1^{st} cav in 2^{nd} cryomodule

ΔE_{calc} = 0.102014 MeV/u

ΔE_{xal} = 0.1019 MeV/u

ΔE_{impact} = 0.10278 MeV/u

<u>b085</u>

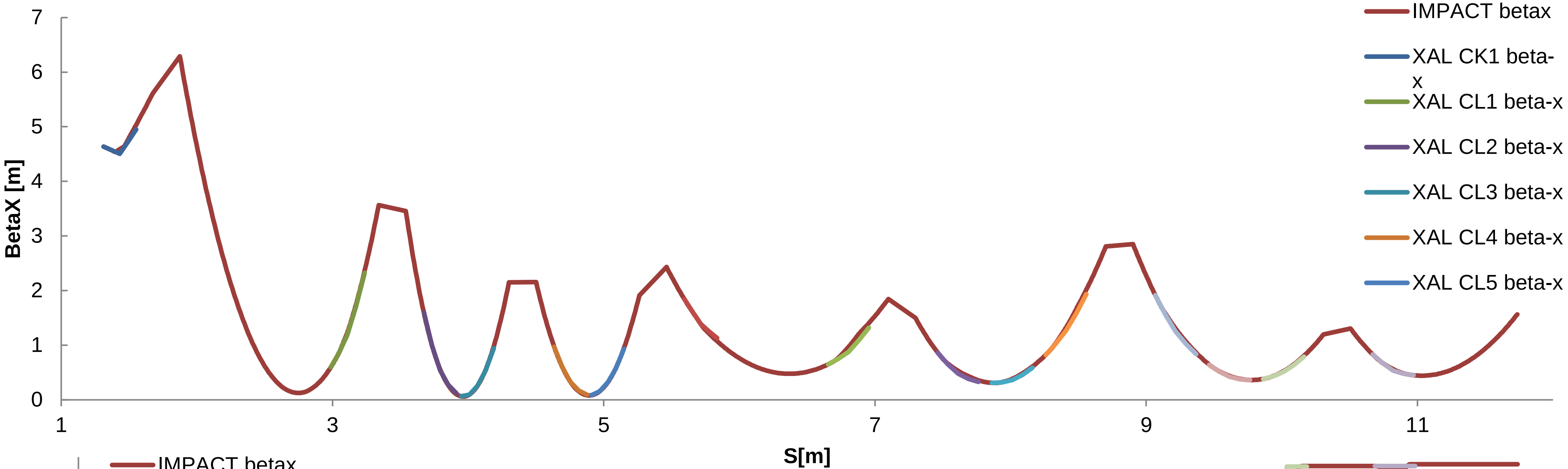
1^{st} cav in 3^{rd} cryomodule

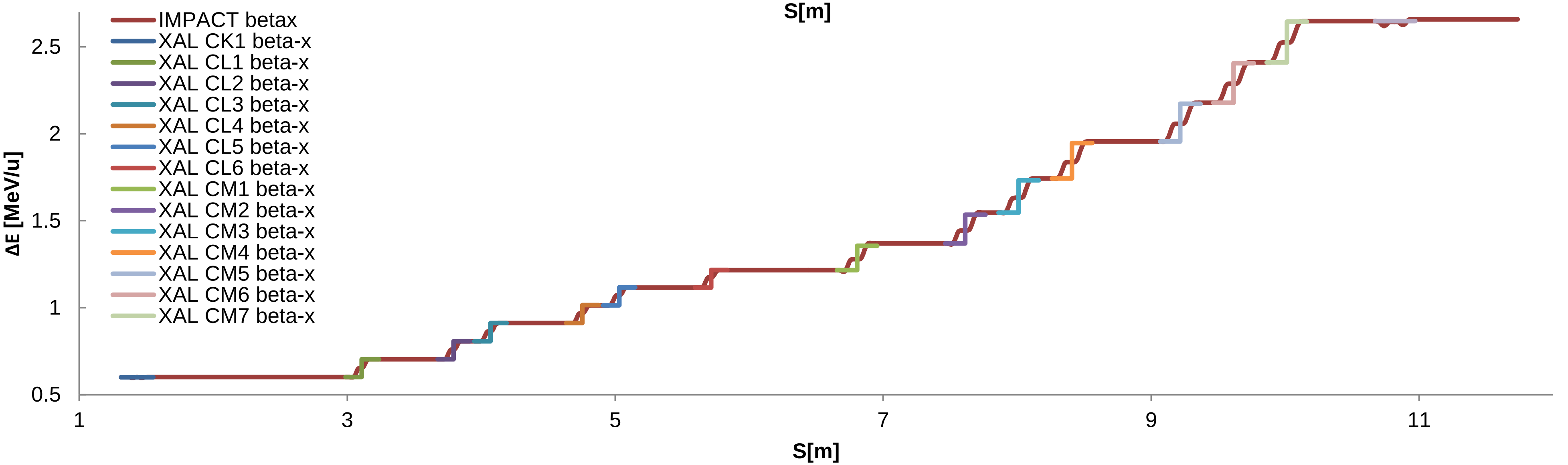
ΔE_{calc} = 0.139382 MeV/u

ΔE_{xal} = 0.139875 MeV/u

ΔE_{impact} = 0.15377 MeV/u

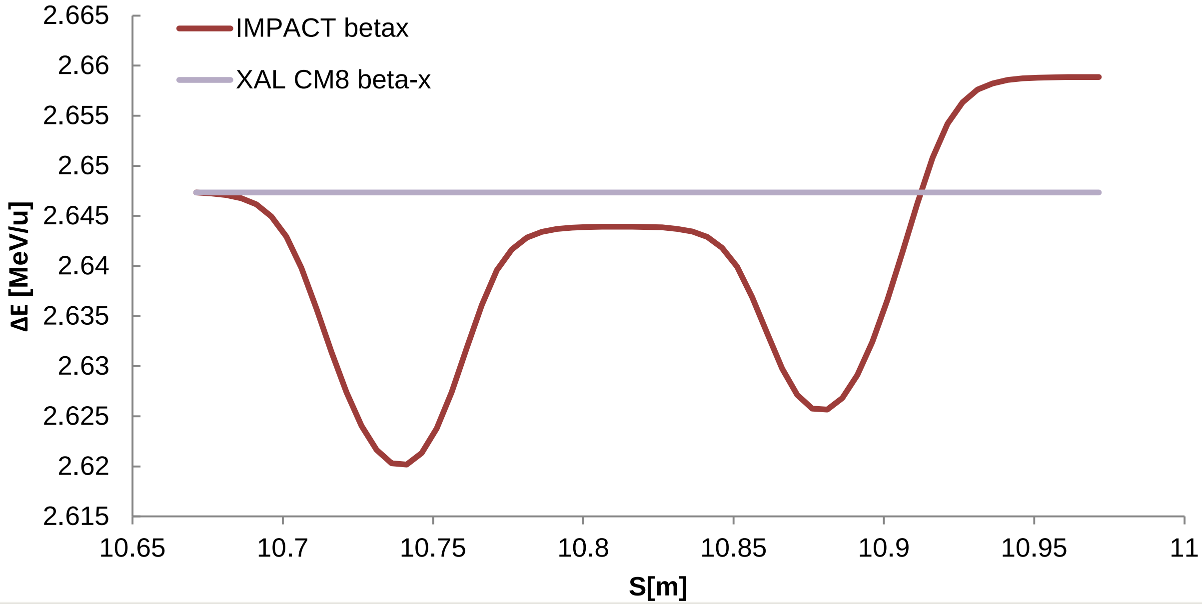
Each Cavity an Individual Sequence





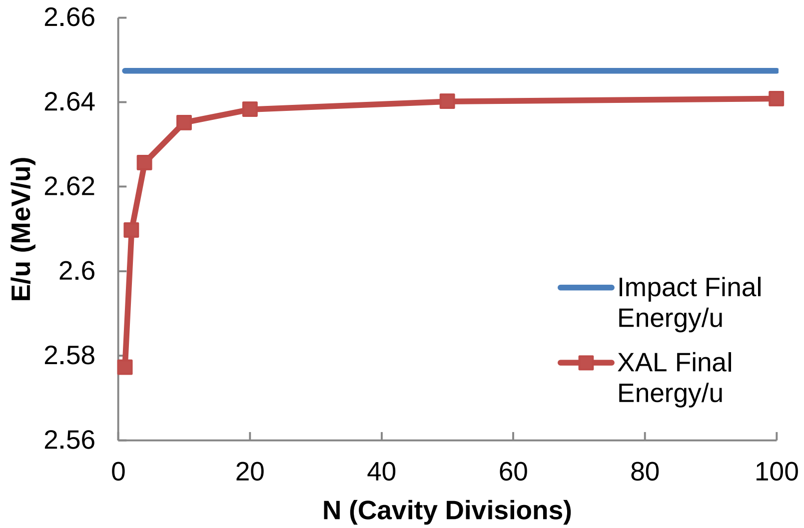
Energy Gain Difference

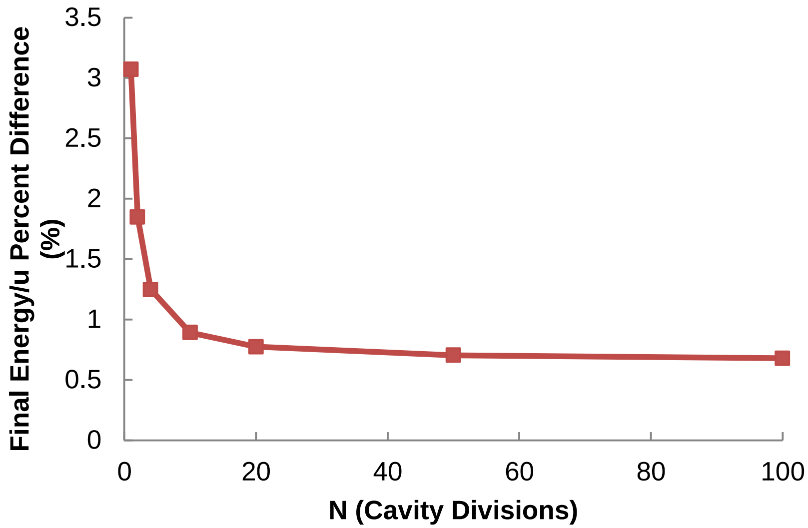
	Position		IMPACT					XAL					IMPACT-XAL				Div by IMPACT	
	Initial	Final	Betai	Betaf	Ei	Ef	ΔE	Betai	Betaf	Ei	Ef	ΔE	Betai diff	Ei diff	Betaf diff	Ef diff	Betaf %diff	Ef %diff
CK1	1.31195	1.55195	4.63602	5.03205	0.6	0.600607	0.000607	4.636	4.9508	0.6	0.6	0	2E-05	0	0.08125	0.000607	1.61465	0.101064
CL1	2.98788	3.23838	0.595215	2.37009	0.600608	0.703387	0.102779	0.5952	2.3226	0.6006	0.702525	0.101925	1.5E-05	8E-06	0.04749	0.000862	2.003721	0.12255
CL2	3.67388	3.91555	1.59544	0.097283	0.703388	0.807186	0.103798	1.5954	0.1154	0.7034	0.80715	0.10375	4E-05	-1.2E-05	-0.01812	3.6E-05	-18.623	0.00446
CL3	3.94888	4.18913	0.06171	0.977426	0.807187	0.910822	0.103635	0.0617	0.9439	0.807175	0.91125	0.104075	1.02E-05	1.2E-05	0.033526	-0.00043	3.430029	-0.04699
CL4	4.63488	4.87655	0.968955	0.082104	0.910823	1.01382	0.102997	0.969	0.0934	0.910825	1.0143	0.103475	-4.5E-05	-2E-06	-0.0113	-0.00048	-13.7579	-0.04735
CL5	4.90988	5.15013	0.082172	0.951077	1.01382	1.11562	0.1018	0.0822	0.9326	1.013825	1.116175	0.10235	-2.8E-05	-5E-06	0.018477	-0.00055	1.942745	-0.04975
CL6	5.59588	5.83616	1.83798	1.07195	1.11562	1.21604	0.10042	1.838	1.1287	1.115625	1.216625	0.101	-2E-05	-5E-06	-0.05675	-0.00059	-5.29409	-0.04811
CM1	6.65516	6.95543	0.64811	1.40394	1.216	1.36981	0.15381	0.6481	1.3196	1.216	1.3558	0.1398	1E-05	0	0.08434	0.01401	6.007379	1.02277
CM2	7.46316	7.76405	0.862895	0.355972	1.36978	1.54665	0.17687	0.8629	0.3248	1.369775	1.534425	0.16465	-5E-06	5E-06	0.031172	0.012225	8.756869	0.790418
CM3	7.86116	8.16205	0.30968	0.608375	1.54662	1.74302	0.1964	0.3097	0.5867	1.546625	1.733	0.186375	-2E-05	-5E-06	0.021675	0.01002	3.56277	0.574864
CM4	8.25916	8.55943	0.821177	2.01728	1.74299	1.95484	0.21185	0.8212	1.9354	1.743	1.947125	0.204125	-2.3E-05	-1E-05	0.08188	0.007715	4.058931	0.394661
CM5	9.06716	9.36805	1.9172	0.861468	1.95481	2.17822	0.22341	1.9172	0.8329	1.9548	2.17265	0.21785	0	1E-05	0.028568	0.00557	3.3162	0.255713
CM6	9.46516	9.76605	0.643846	0.36175	2.17819	2.40994	0.23175	0.6438	0.3577	2.1782	2.406075	0.227875	4.6E-05	-1E-05	0.00405	0.003865	1.119558	0.160377
CM7	9.86316	10.1634	0.38057	0.803766	2.40992	2.64737	0.23745	0.3806	0.788	2.409925	2.64485	0.234925	-3E-05	-5E-06	0.015766	0.00252	1.961516	0.095189
CM8	10.6712	10.9715	0.827893	0.4492	2.64733	2.65885	0.01152	0.8279	0.4487	2.647325	2.647325	0	-7E-06	5E-06	0.0005	0.011525	0.111309	0.433458

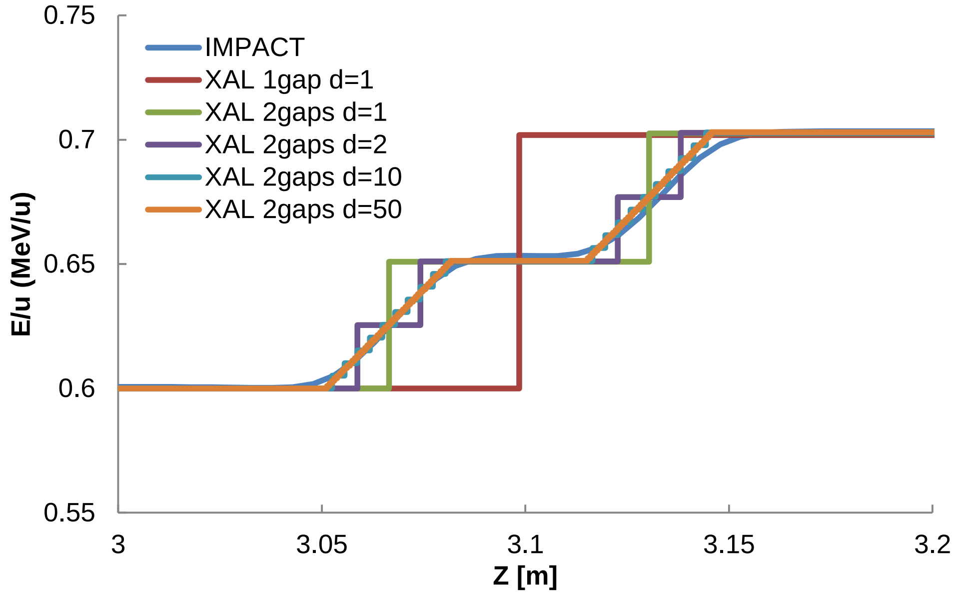


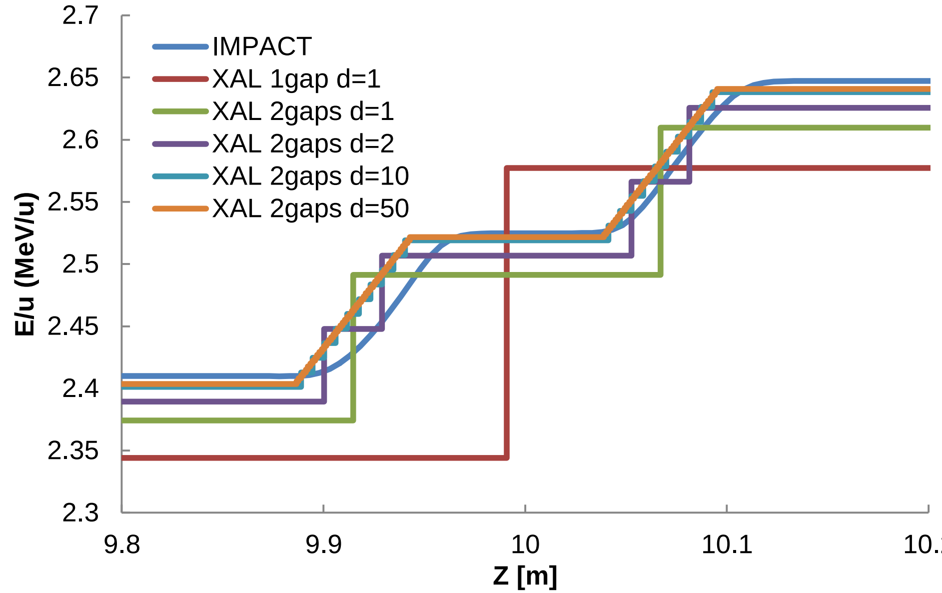
CL Avg %diff	CM Avg %diff
0.053200187	**0.465931382**

Cavity Divisions

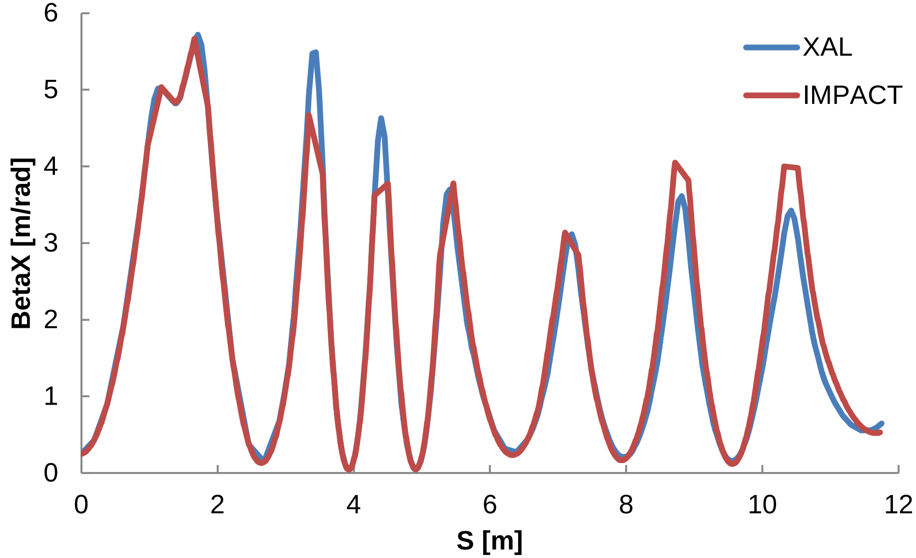


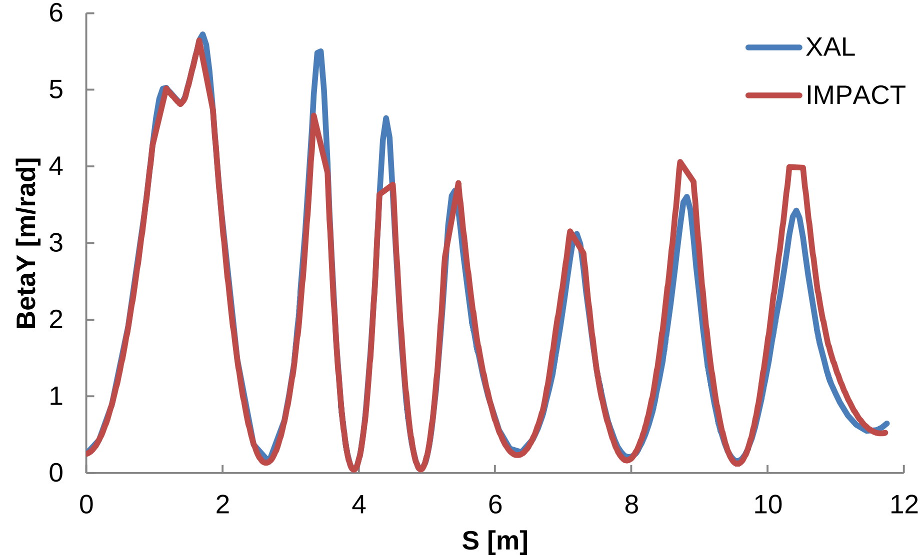


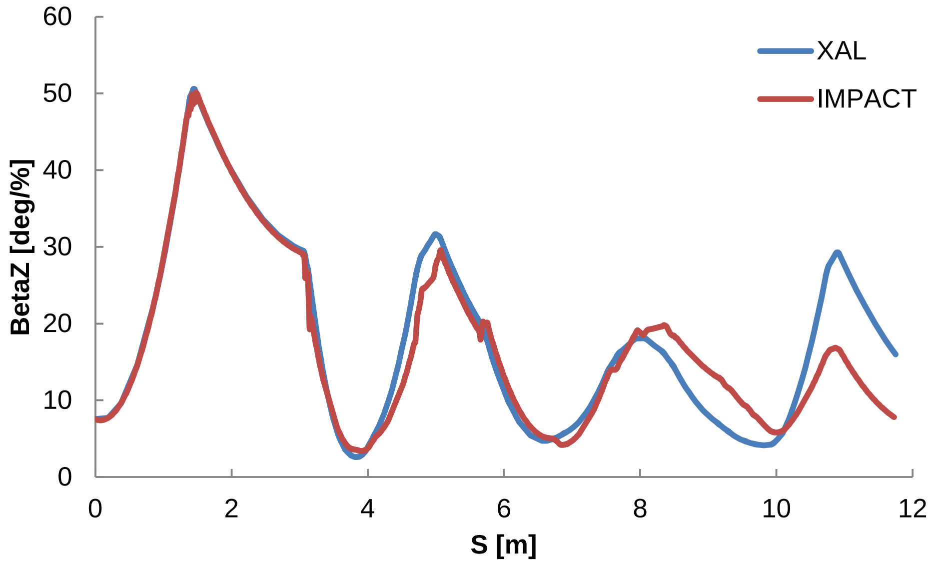


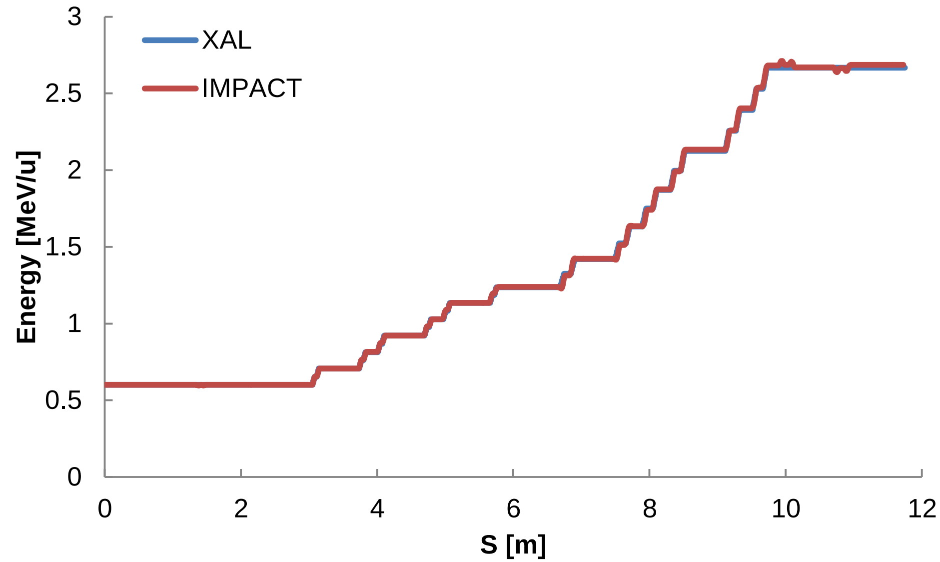


Split to 20 Gap Divisions

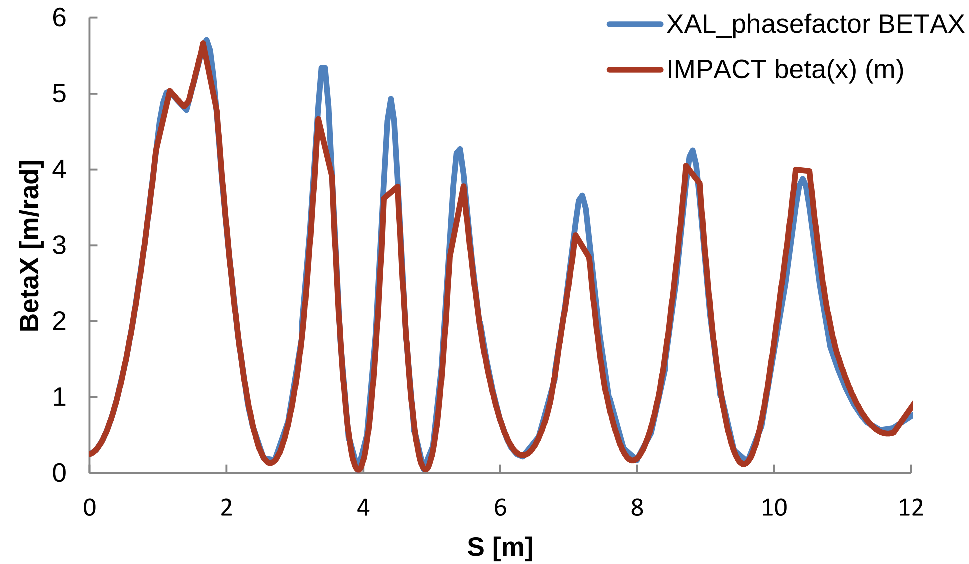


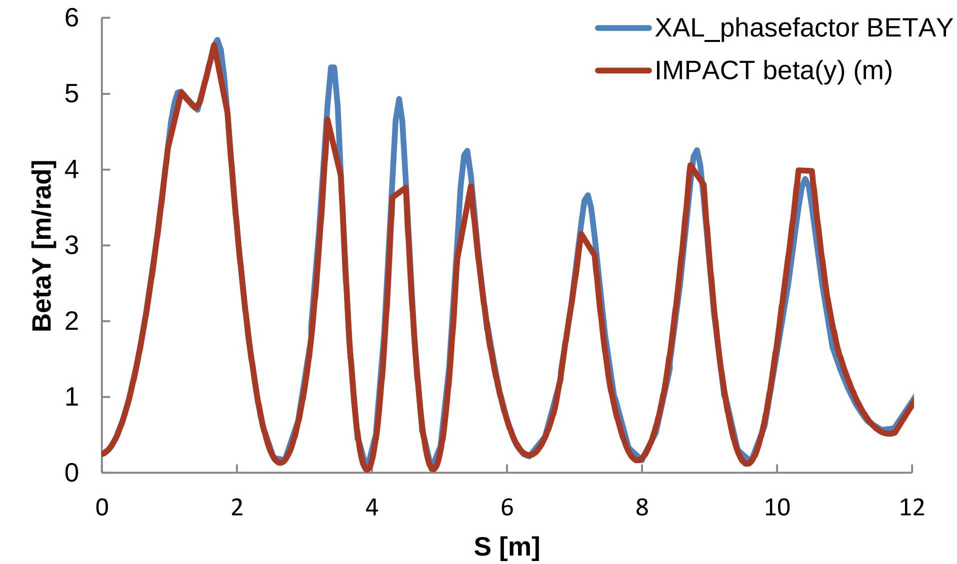


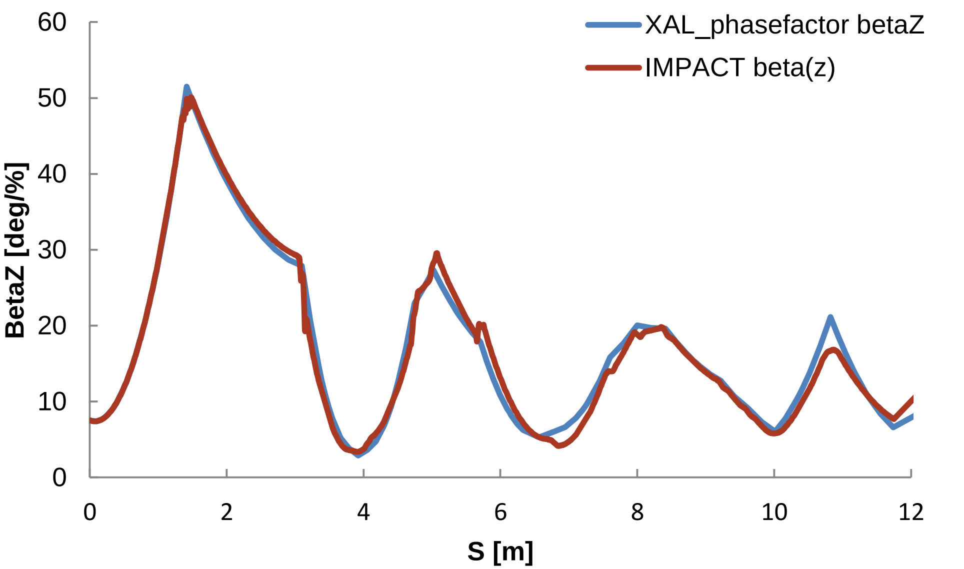


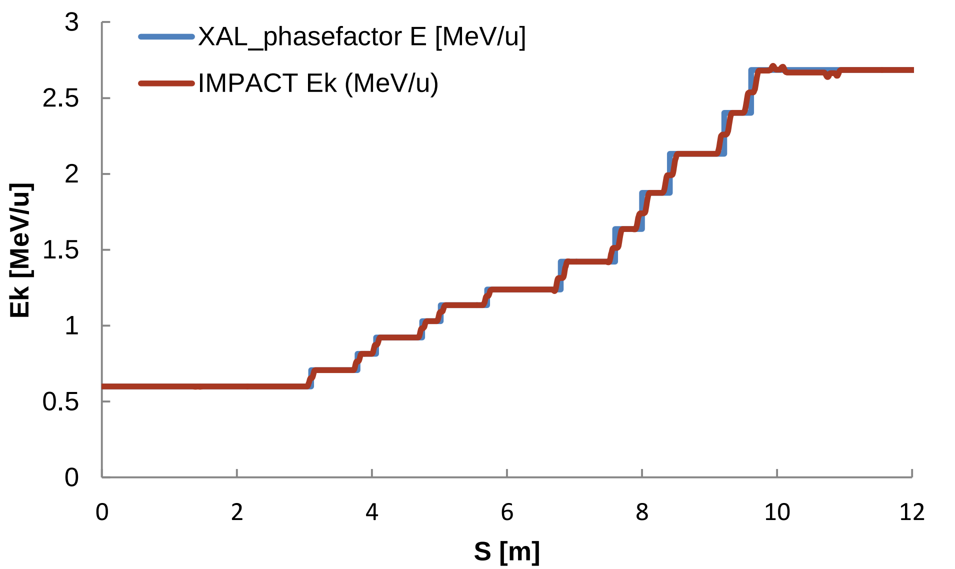


XAL Change Cavity Phase









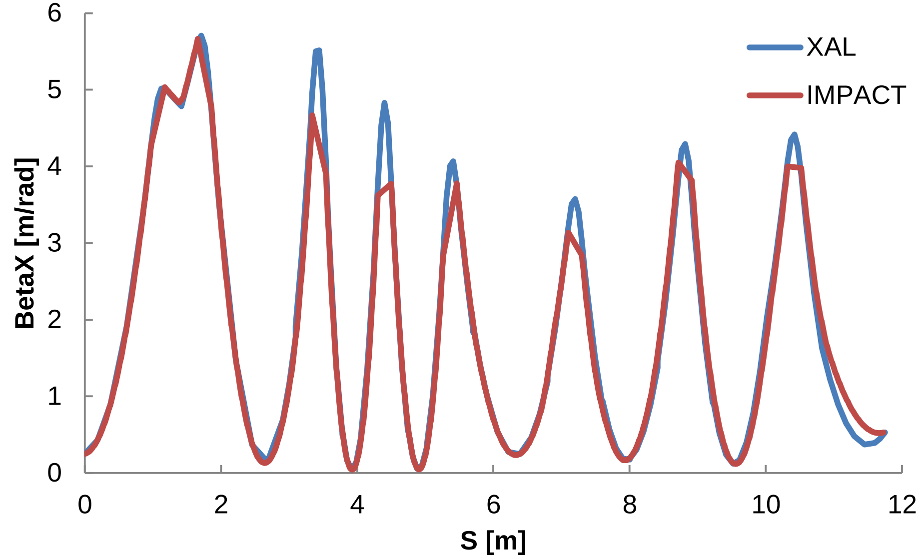
Conclusions on Cavity Benchmark

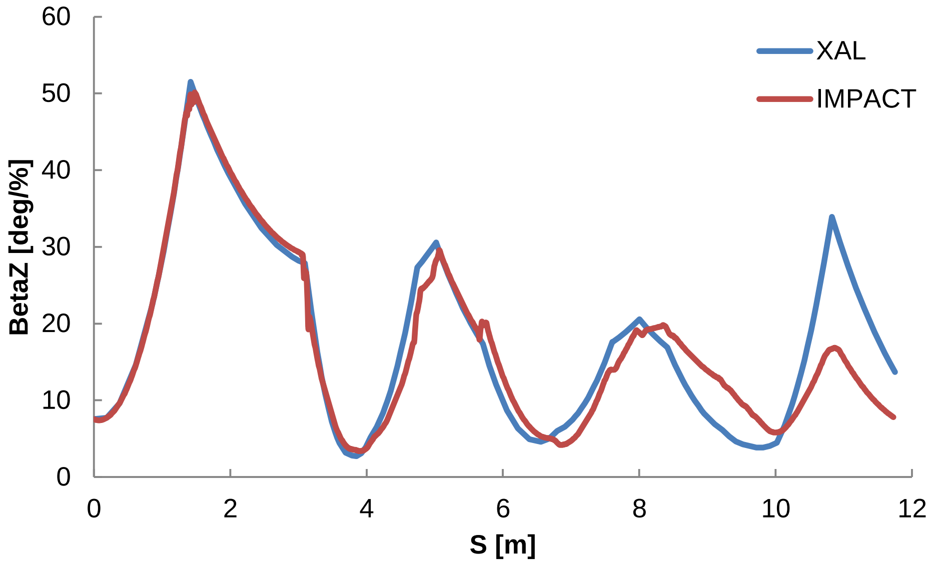
- One gap model is valid in regions where TTF curve does not change rapidly

- Two gap model needs to be explored
    - Phase advance
    - Transverse/Longitudinal

- Dividing cavity gaps can reproduce energy gain, but not betaZ (phase?)

- Be aware of code differences, ie re-bunching cavities may not match IMPACT



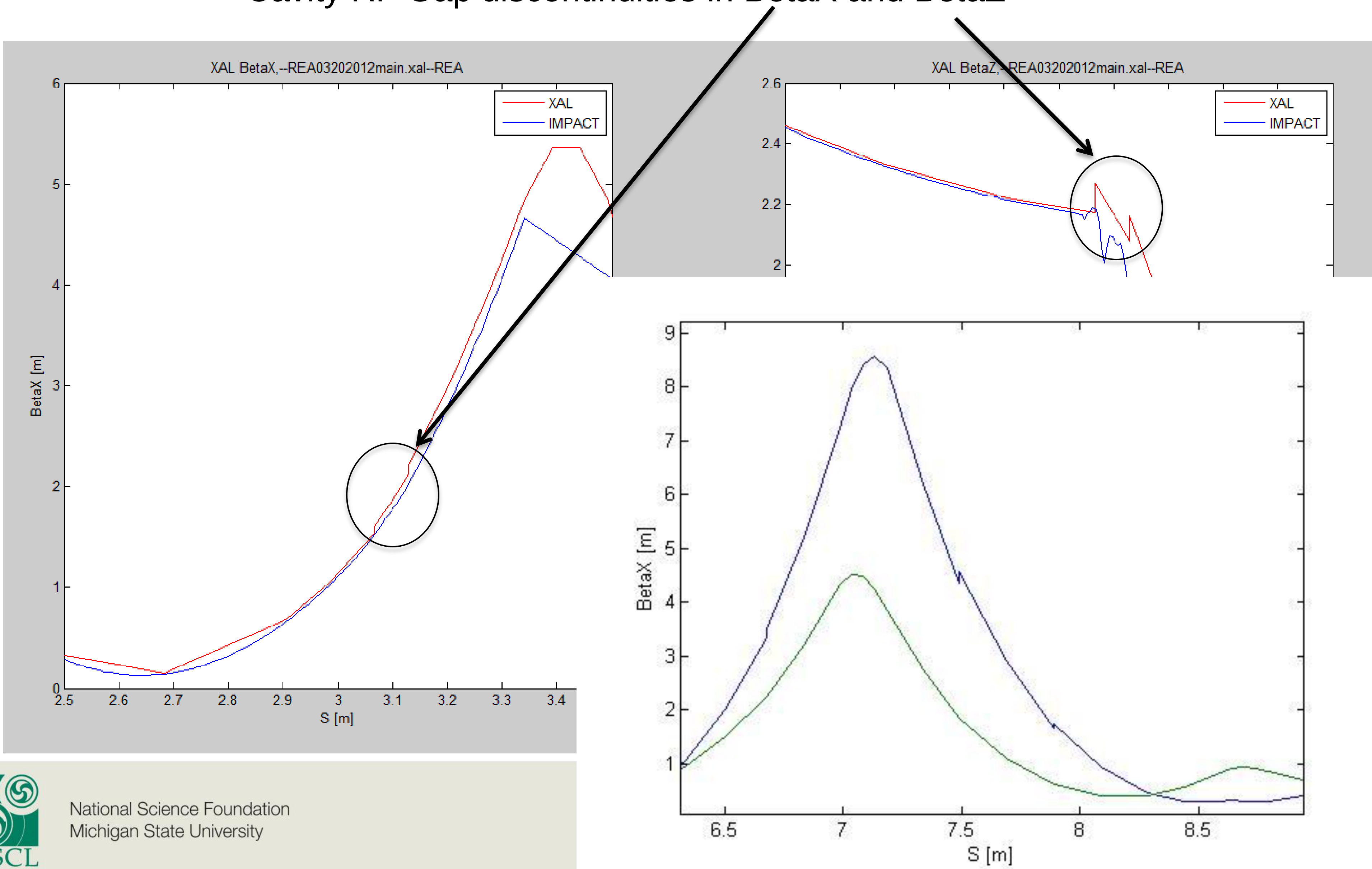


Outline

- ✓ Model Benchmarking Process
- ✓ Benchmarked Elements and Examples
- ✓ Cavity Benchmarking Experience
- ❑ Some Ongoing Questions

Current Issues (1)

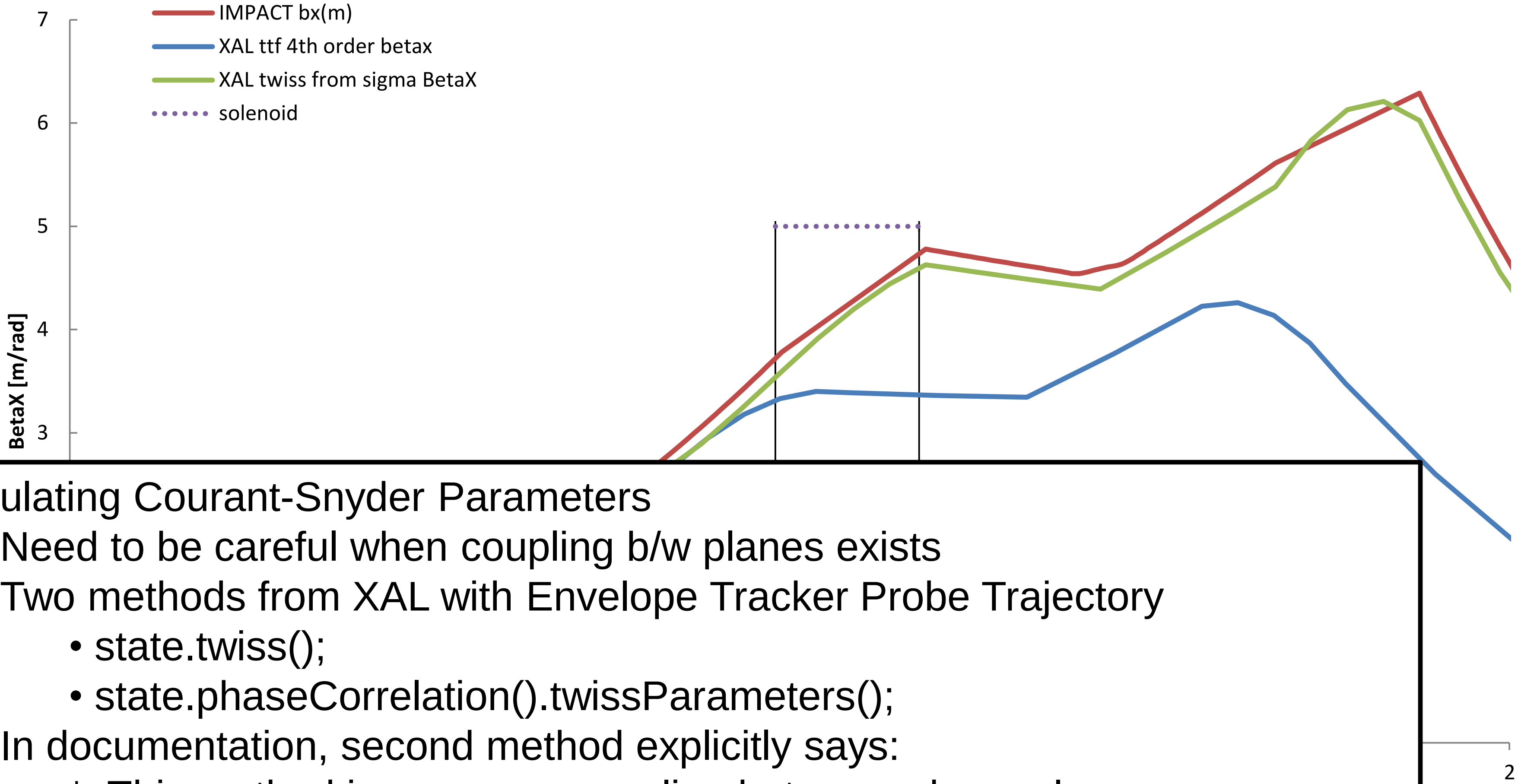
Cavity RF Gap discontinuities in BetaX and BetaZ



National Science Foundation
Michigan State University

Current Issues (2)

ReA3 Linac getBeta (two methods)



- Calculating Courant-Snyder Parameters
  - Need to be careful when coupling b/w planes exists
  - Two methods from XAL with Envelope Tracker Probe Trajectory
    - state.twiss();
    - state.phaseCorrelation().twissParameters();
  - In documentation, second method explicitly says:
    - * This method ignores any coupling between phase planes.
    - * **TODO - Make the method consider the general case of**
    - * **coupling between phase planes** and return the Twiss parameters
    - * as projections that one would observe in experiments.

However, this seems to be the method that agrees with IMPACT

Current Issues (3)

- Documentation Improvement
  - Units
  - Definitions (phaseFactor, polarity, etc.)
  - Workarounds

- Vertical Bend Elements
  - Sph/Cyl bend working on a case-by-case basis

- Some functions may not be working (may just be in Matlab)
  - EnvelopeTrajectory.getStates()
  - buildCorrelation()
  - …

- Solenoid model
  - Is hard-edge a good enough approximation? (May need L_{eff})

- Need to benchmark
  - Offsets (misalignment)
  - Chromaticity
  - …

Outline

✓ Model Benchmarking Process

✓ Benchmarked Elements and Examples

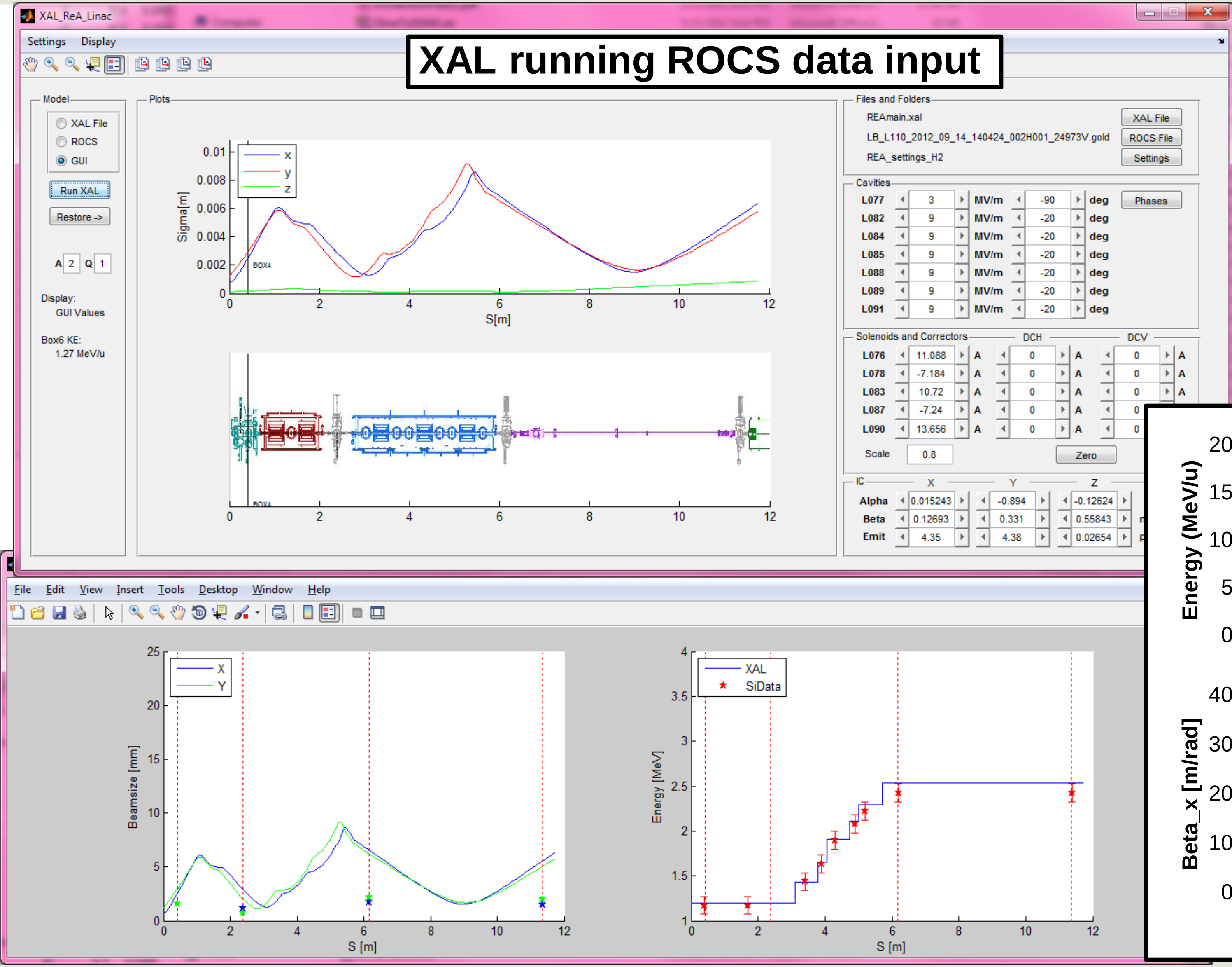
✓ Cavity Benchmarking Experience

✓ Some Ongoing Questions

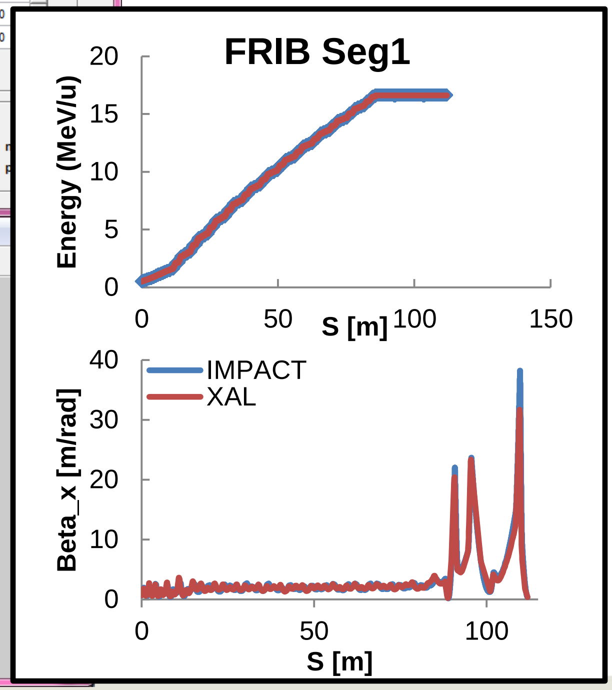
❑ Outlook

Outlook

XAL running ROCS data input

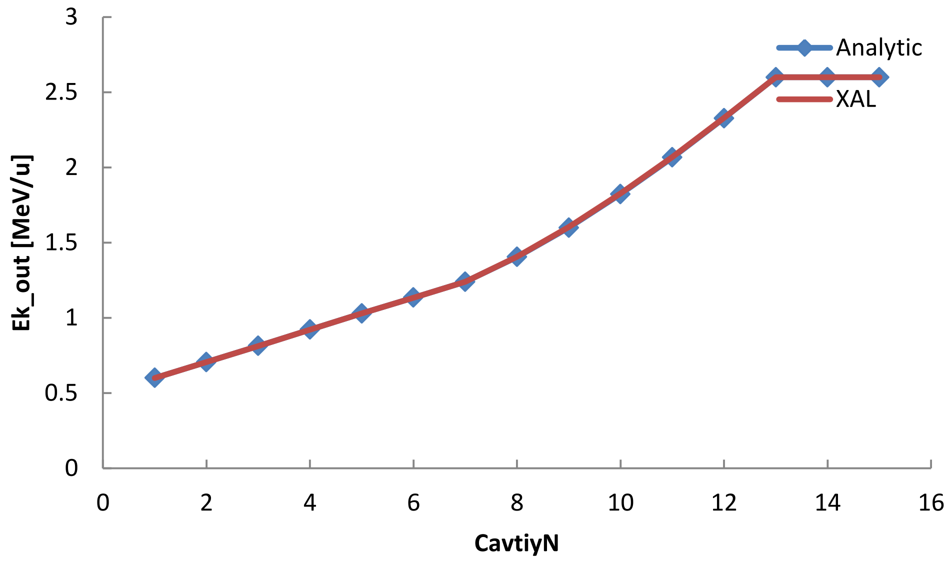


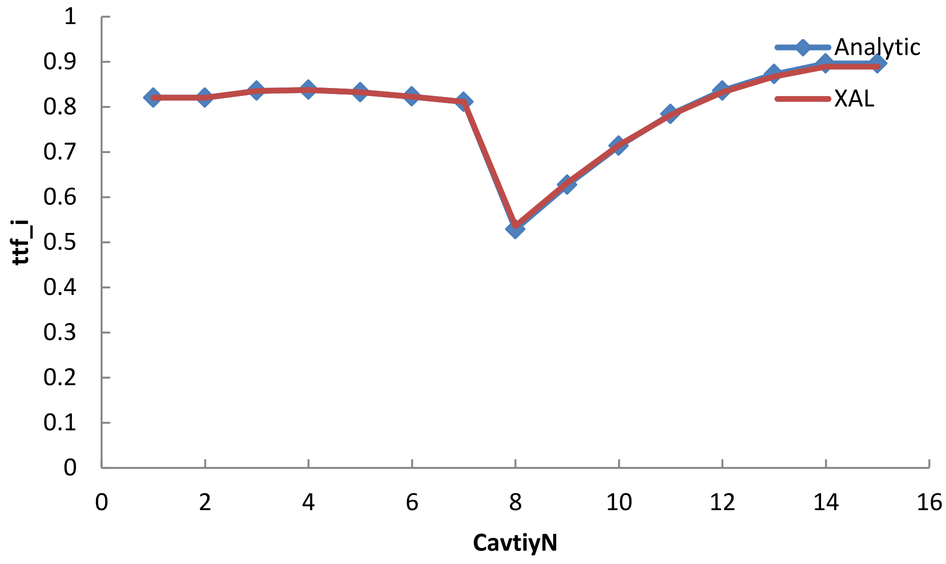
- Initial benchmarking of XAL is completed for using XAL as a tool at ReA3 and FRIB
- Still have work to be done (misalignment, chromaticity etc.)

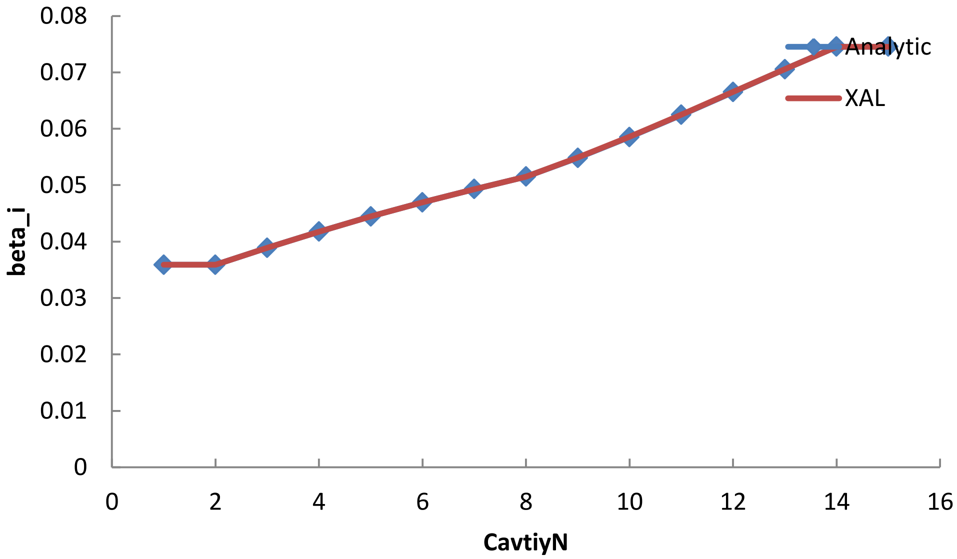


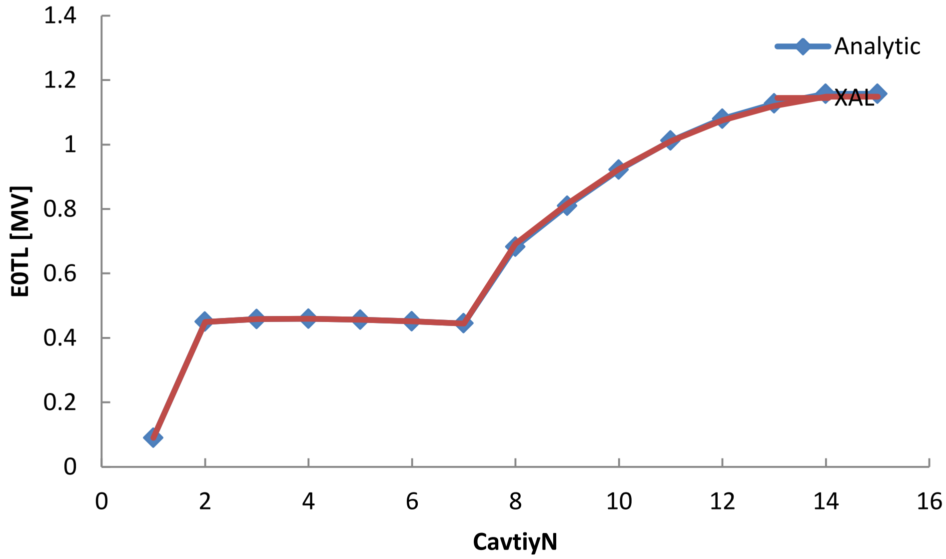
Extra Slides

Cavity Benchmark









Cavity Benchmark

%Diff--Analytic, XAL w/o extra factor

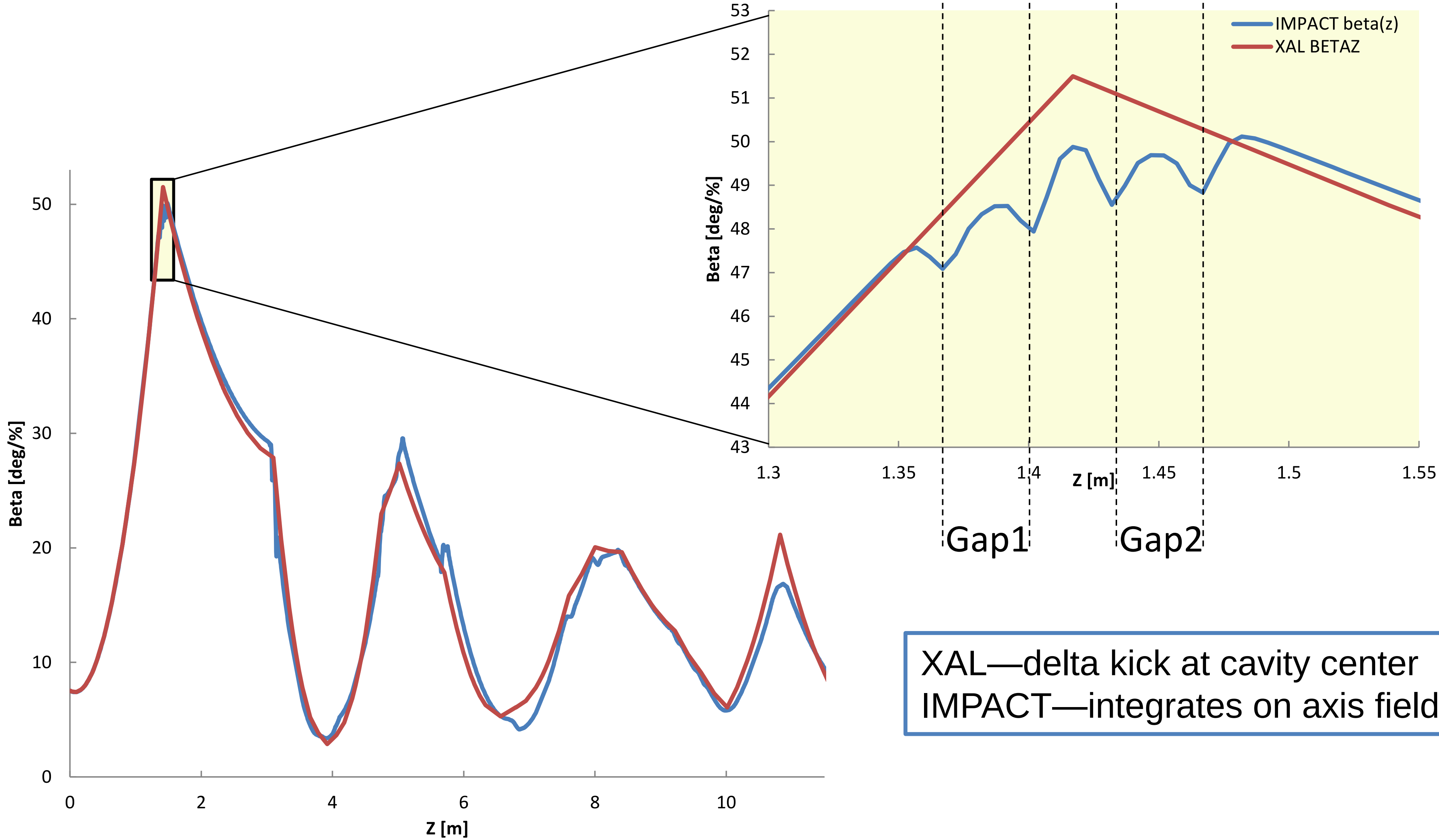
mx21	mx22	mz21	mz22	kt	kz	Ek_i MeV/u	Ek_out MeV/u	beta_i	beta_out	g_i	g_out	ttf_i	EoTL MV
-0.00521	0	-0.005210882	0	-0.00552	-0.00552	0	0	-0.00031	-0.00031	-2.6E-06	-2.6E-06	-0.00614	-0.00614
0.160146	0.000461	0.160144905	0.000460676	0.159383	0.159382	0	-0.00092	-0.00031	-0.00076	-2.6E-06	1.29E-06	-0.00614	-0.00614
0.125192	0.000169	0.125186787	0.000169427	0.124257	0.124255	-0.00092	-0.00126	-0.00076	-0.00093	1.29E-06	-5.1E-06	-0.00348	-0.00348
0.096621	0.000138	0.096615708	0.000137803	0.095547	0.095545	-0.00126	-0.00154	-0.00093	-0.00107	-5.1E-06	-7.5E-07	-0.00362	-0.00362
0.078166	-3.3E-05	0.078162211	-3.31607E-05	0.077129	0.077121	-0.00154	-0.00146	-0.00107	-0.00104	-7.5E-07	-1.4E-06	-0.0009	-0.0009
0.062938	-5.1E-05	0.062931955	-5.10903E-05	0.061949	0.061944	-0.00146	-0.00137	-0.00104	-0.00099	-1.4E-06	4.85E-07	-0.00036	-0.00036
0.05071	-2.1E-05	0.050705722	-2.05392E-05	0.049744	0.049741	-0.00137	-0.00132	-0.00099	-0.00096	4.85E-07	-5.2E-06	-0.00087	-0.00088
0.096642	0.000185	0.096634392	0.000185162	0.095491	0.095483	-0.00132	-0.0017	-0.00096	-0.00115	-5.2E-06	-6.5E-06	-0.00526	-0.00448
0.101436	0.000487	0.101427581	0.000487198	0.099797	0.099791	-0.0017	-0.00267	-0.00115	-0.00163	-6.5E-06	-9.8E-06	-0.01043	-0.00965
0.108308	3.13E-05	0.108295374	3.12857E-05	0.106641	0.106627	-0.00267	-0.00273	-0.00163	-0.00167	-9.8E-06	-2.9E-06	-0.00395	-0.00318
0.101578	1.27E-05	0.101562889	1.26924E-05	0.099898	0.099881	-0.00273	-0.00276	-0.00167	-0.00168	-2.9E-06	-7.7E-06	-0.00372	-0.00294
0.092772	-0.00011	0.092758038	-0.000109633	0.091202	0.091184	-0.00276	-0.00253	-0.00168	-0.00157	-7.7E-06	-4.5E-06	-0.00157	-0.00079
0.080368	-7.3E-05	0.080354508	-7.26285E-05	0.078874	0.078856	-0.00253	-0.00239	-0.00157	-0.00149	-4.5E-06	-4.3E-06	-0.00192	-0.00115
0.004039	0	0.004024065	0	0.002538	0.002521	-0.00239	-0.00239	-0.00149	-0.00149	-4.3E-06	-4.3E-06	-0.00126	-0.00048
0.004039	0	0.004024065	0	0.002538	0.002521	-0.00239	-0.00239	-0.00149	-0.00149	-4.3E-06	-4.3E-06	-0.00126	-0.00048

$$\beta_{avg} = \sqrt{1 - \frac{1}{\gamma_{avg}}}$$

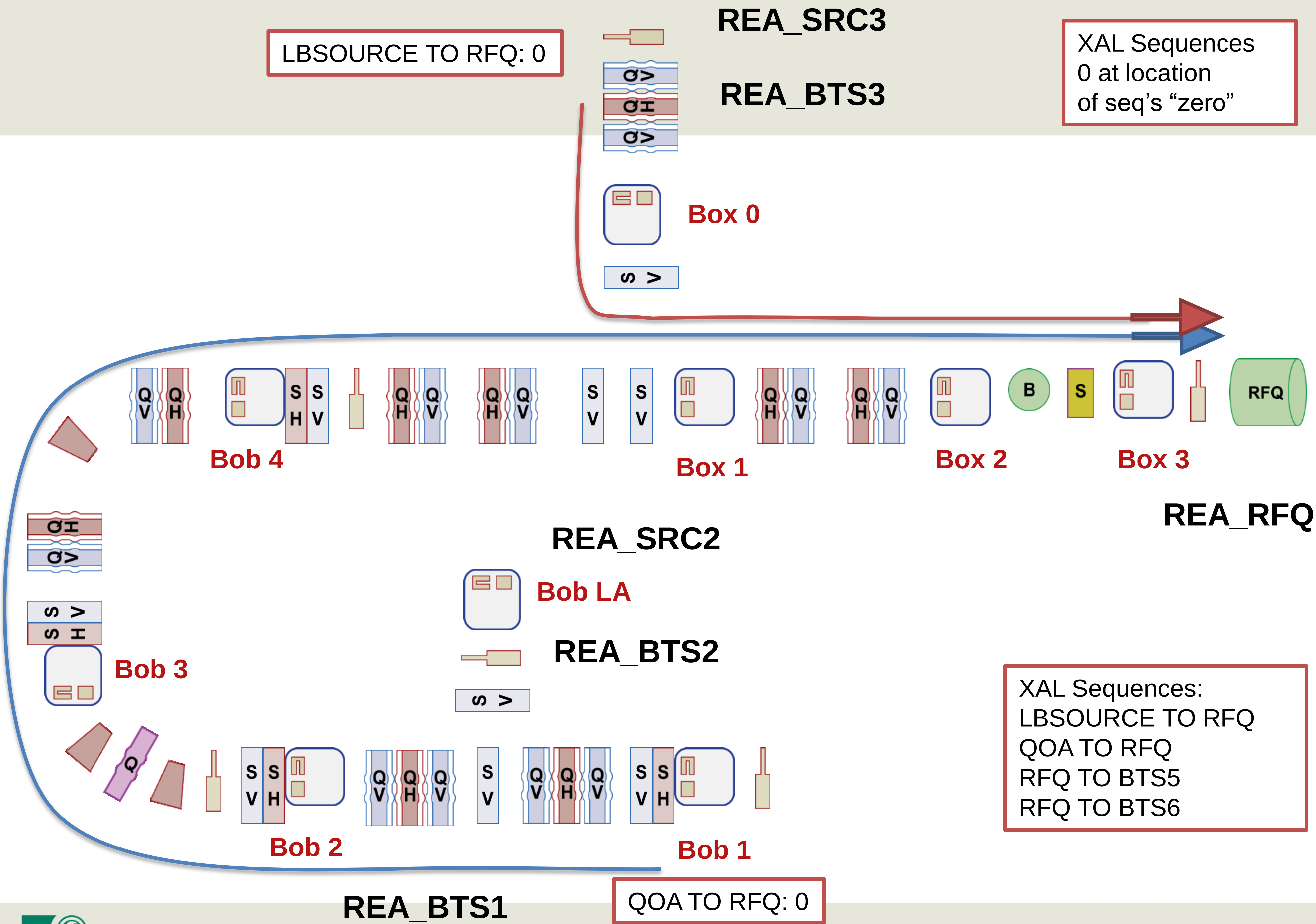
$$\beta_{avg} \neq \frac{\beta_i + \beta_f}{2}$$

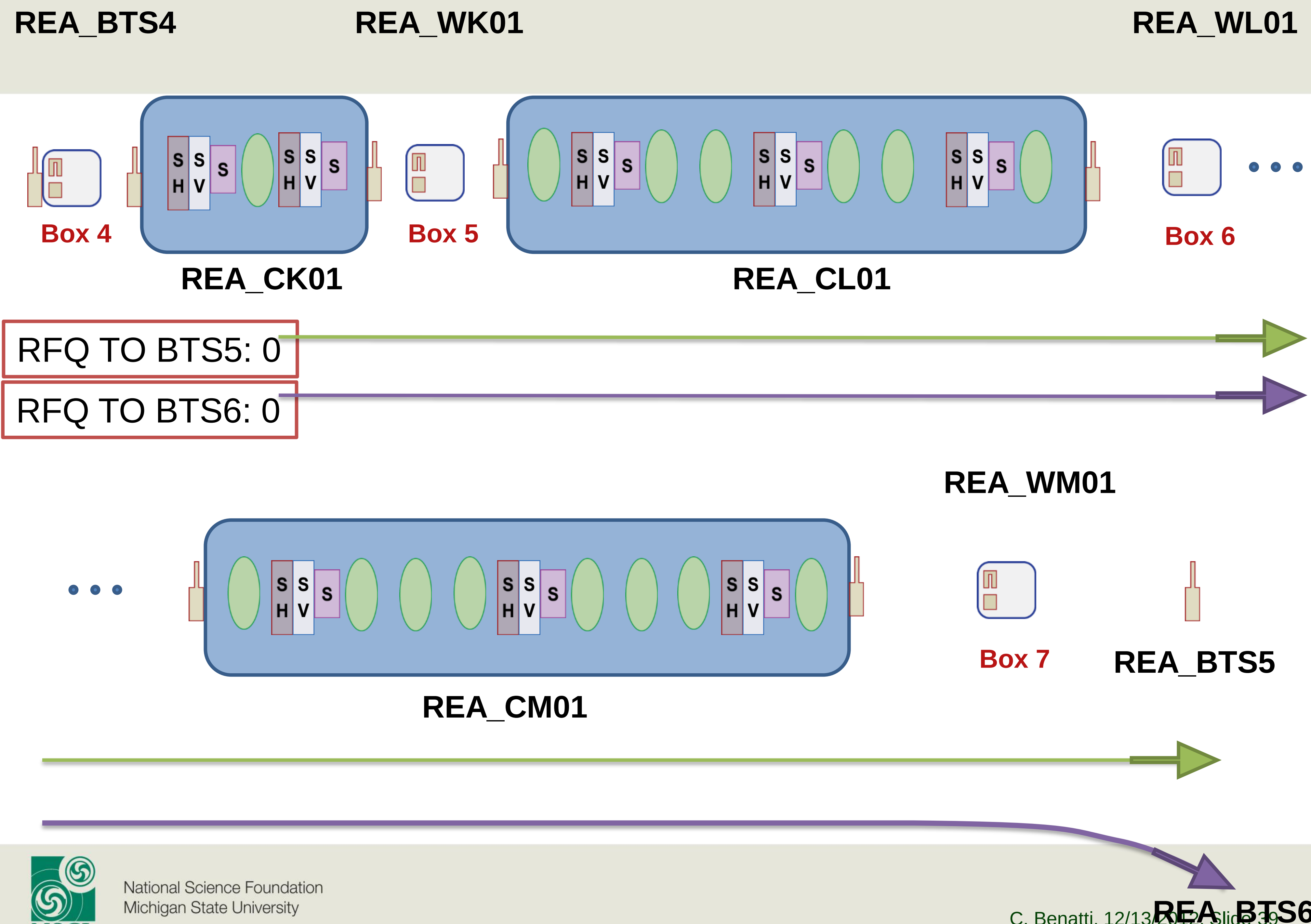
N	mx21/kx	mz21/kz	beta_avg	gamma_avg	kt*(bgav)^2
1	0.000306	0.000305	-0.00031	-2.6E-06	-0.00613233
2	0.000765	0.000764	-0.08285	-6.7E-07	-0.00612737
3	0.000936	0.000933	-0.06392	-1.9E-06	-0.00346508
4	0.001075	0.001072	-0.04961	-2.9E-06	-0.00360445
5	0.001038	0.001042	-0.03903	-1.1E-06	-0.00087817
6	0.00099	0.000988	-0.03116	-4.6E-07	-0.00034429
7	0.000966	0.000965	-0.02531	-2.4E-06	-0.00085772
8	0.001152	0.001152	-0.05001	-5.8E-06	-0.00446975
9	0.00164	0.001638	-0.05475	-8.2E-06	-0.00964705
10	0.001669	0.00167	-0.05494	-6.4E-06	-0.00315721
11	0.001682	0.001683	-0.05144	-5.3E-06	-0.00292053
12	0.001572	0.001575	-0.04601	-6.1E-06	-0.00077177
13	0.001495	0.0015	-0.04002	-4.4E-06	-0.00111797
14	0.001501	0.001503	-0.00149	-4.3E-06	-0.00045683
15	0.001501	0.001503	-0.00149	-4.3E-06	-0.00045683

ReA Cavity Model—One Gap



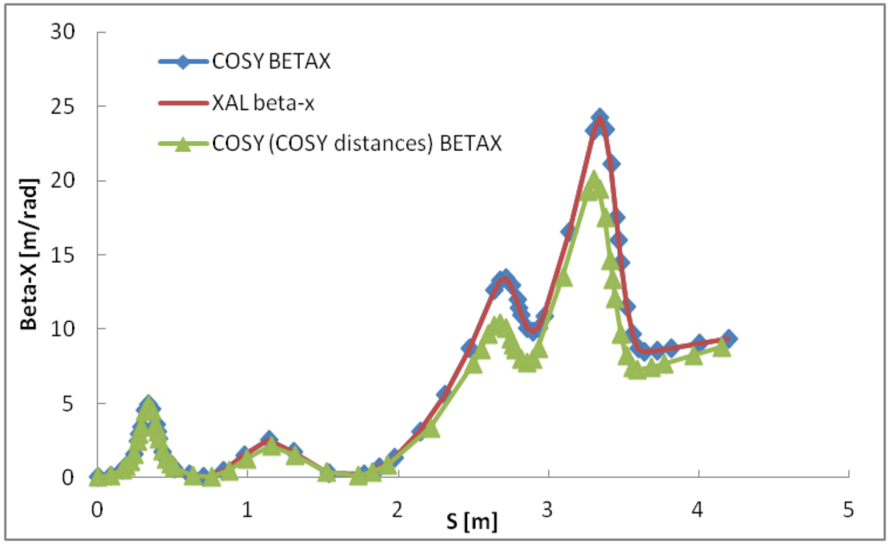
XAL—delta kick at cavity center
IMPACT—integrates on axis field

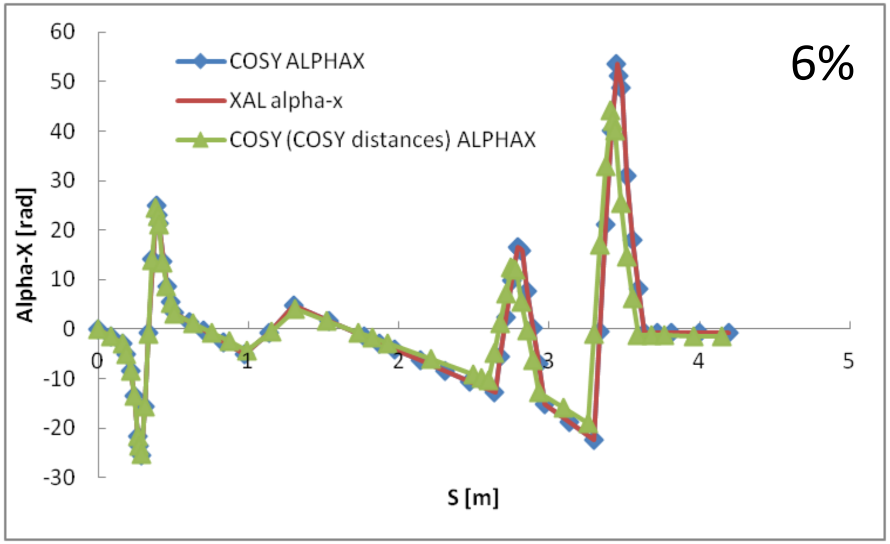

LBSOURCE TO RFQ: 0
REA_SRC3
REA_BTS3
XAL Sequences
0 at location
of seq's "zero"
Box 0
RFQ
Bob 4
Box 1
Box 2
Box 3
REA_RFQ
REA_SRC2
Bob LA
REA_BTS2
Bob 3
XAL Sequences:
LBSOURCE TO RFQ
QOA TO RFQ
RFQ TO BTS5
RFQ TO BTS6
Bob 2
Bob 1
REA_BTS1
QOA TO RFQ: 0

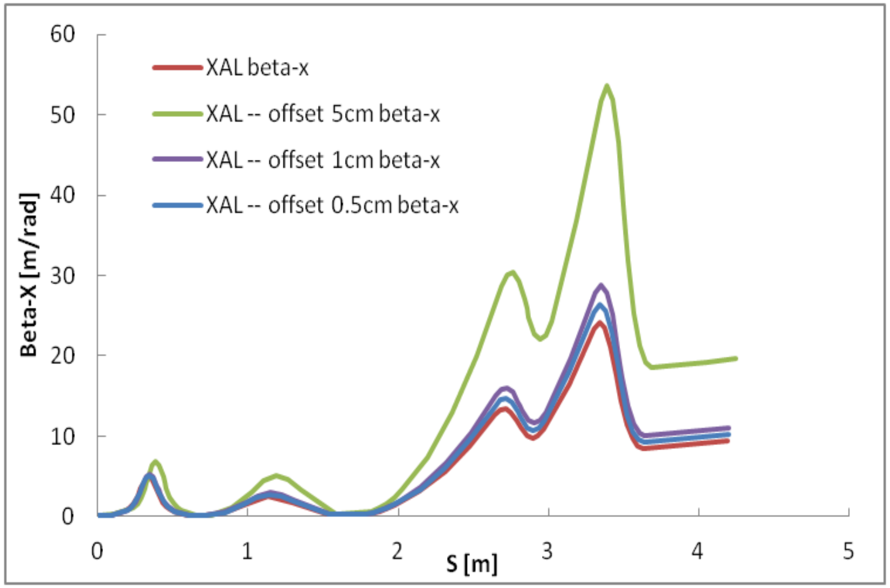


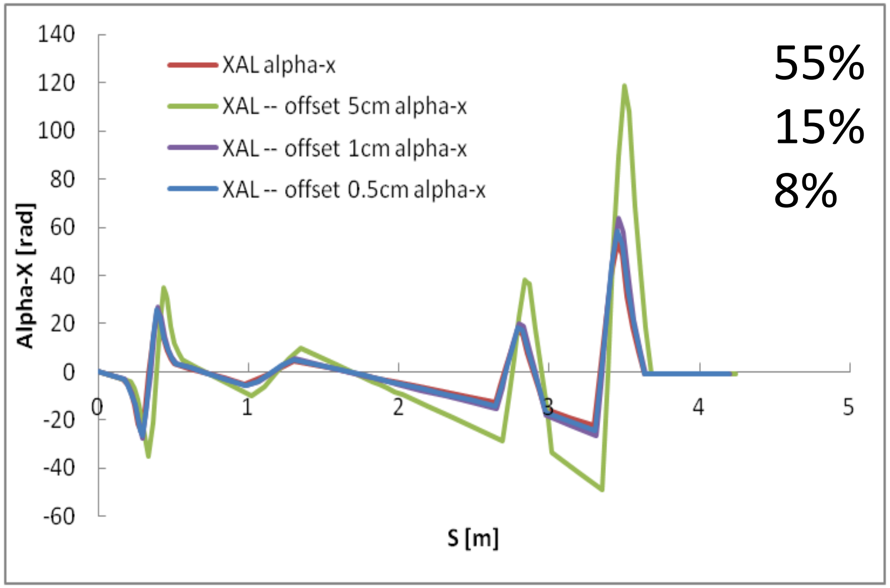
REA_BTS4
REA_WK01
REA_WL01
S H
S V
S
Box 4
REA_CK01
Box 5
REA_CL01
Box 6
RFQ TO BTS5: 0
RFQ TO BTS6: 0
REA_WM01
REA_CM01
Box 7
REA_BTS5
REA_BTS6

Distance Mismatch Study









Energy gain, TTF, gradient

Energy gain: $$\Delta W_p = q\int_{-L/2}^{L/2} E_z(z_p,t)dz_p$$

In a resonator $E_z(r,z,t)=E_z(r,z)\cos(\omega t+\varphi)$. (For simplicity, we assume to be on axis so that $r=0$, and $E_z(0,z) \equiv E_z(z)$).

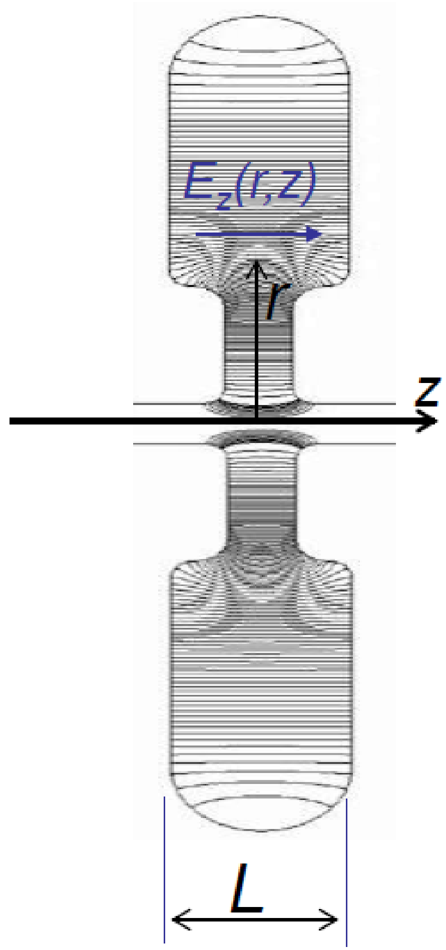
A particle with velocity βc, which crosses $z=0$ when $t=0$, sees a field $E_z(z)\cos(\omega z/\beta c+\varphi)$.

Transit time factor: $$T(\beta) = \frac{\int_{-L/2}^{L/2} E_z(z)\cos\left(\frac{\omega z}{\beta c}\right)dz}{\int_{-L/2}^{L/2} E_z(z)dz}$$

Avg. accelerating field: $$E_a = \frac{1}{L}\int_{-L/2}^{L/2} E_z(z)dz$$

We obtain a simple espression for the energy gain

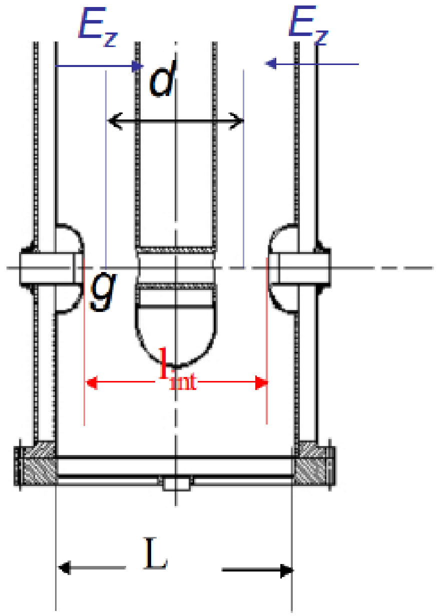
$$\Delta W_p = qE_aLT(\beta)\cos\varphi$$

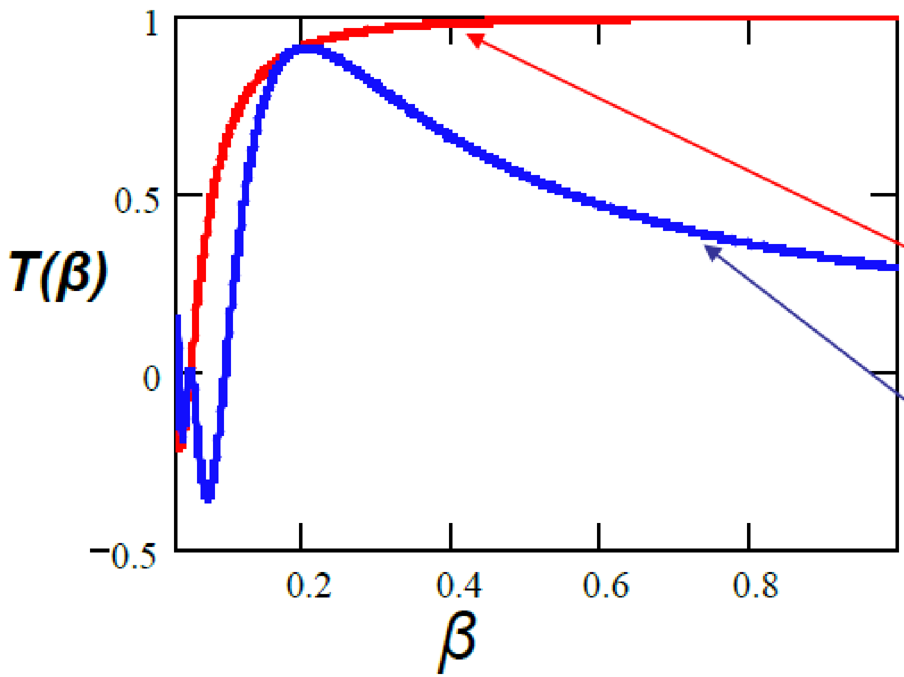


T(β) for 2 gap (π mode)

(constant E_z approximation)

$$T(\beta) \cong \frac{\sin\left(\frac{\pi g}{\beta\lambda}\right)}{\left(\frac{\pi g}{\beta\lambda}\right)} \sin\left(\frac{\pi d}{\beta\lambda}\right)$$





1° term: 1-gap effect → *g<βλ/2*

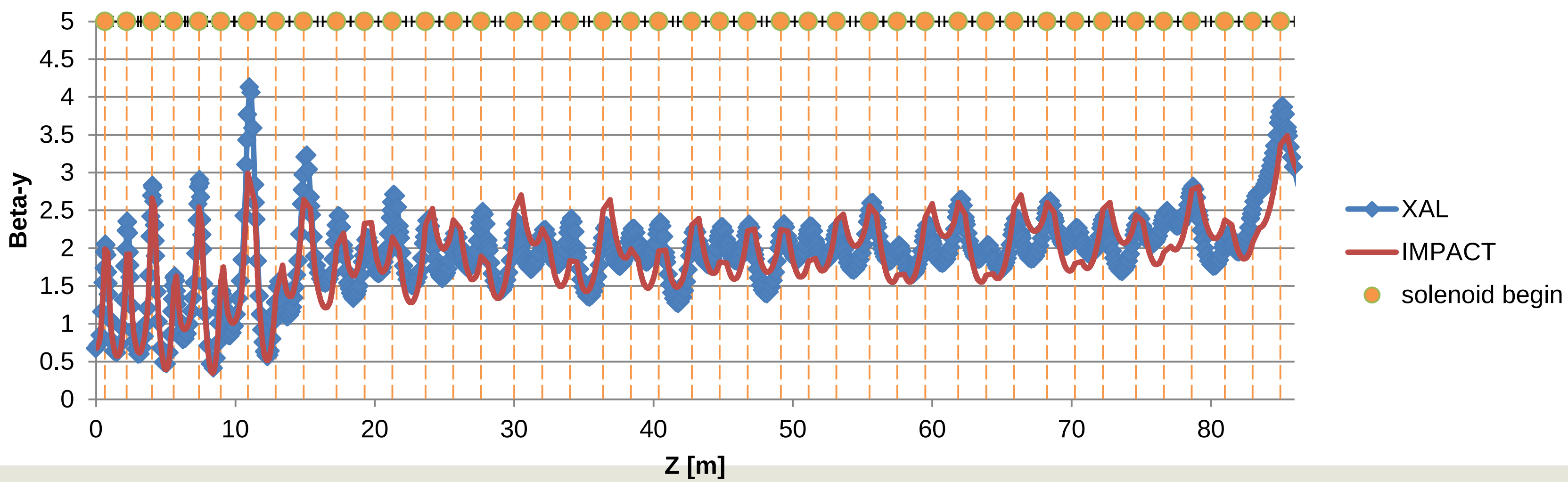
2° term: 2 gap effect → *d~βλ/2*

1°+ 2° term TTF curve

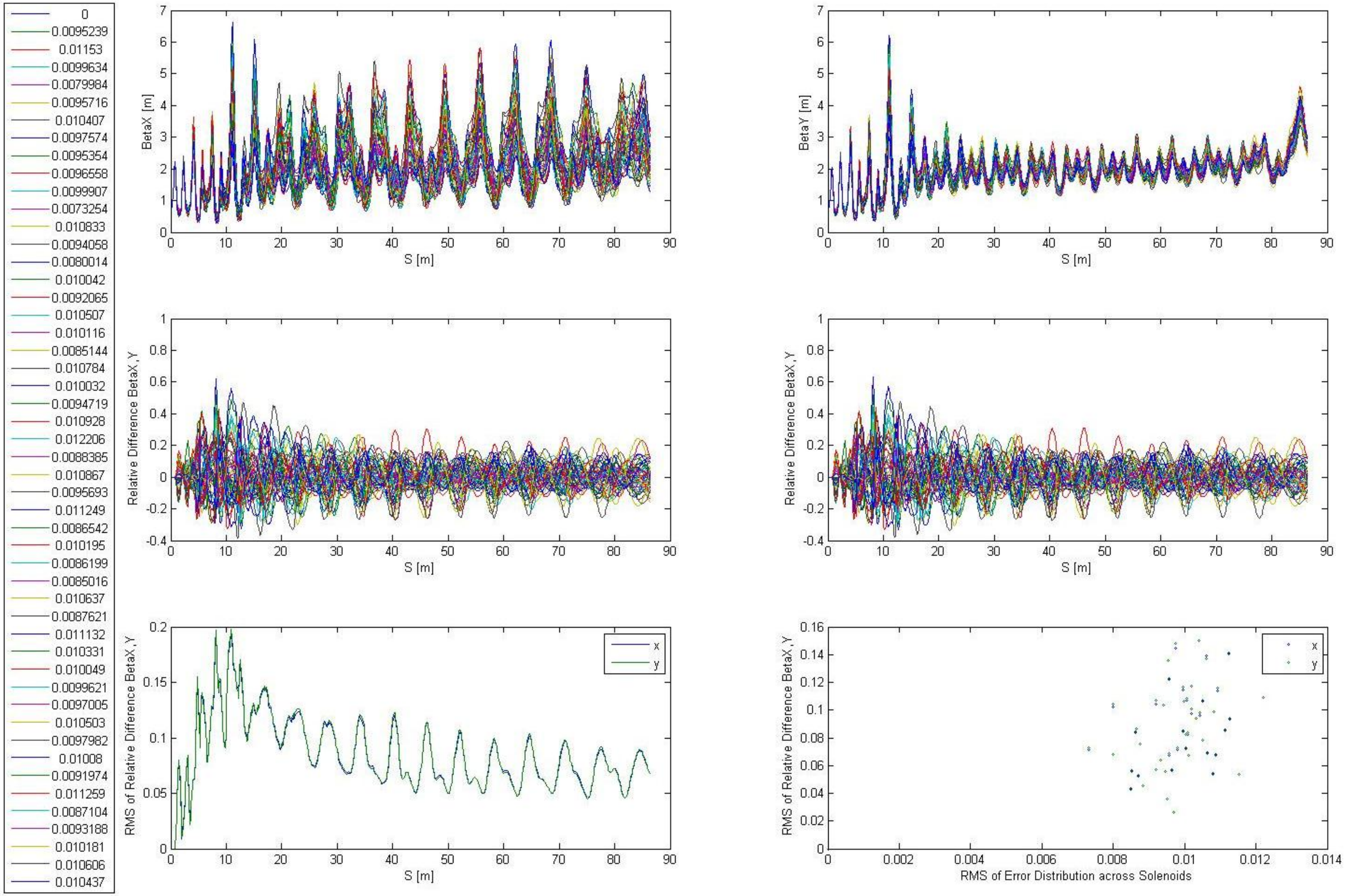
(For more than 2 equal gaps in π mode, the formulas change only in the 2° term)

Solenoid Error Study

- XAL Model running through Matlab
- FRIB Seg1 Linac only
- Introduce Gaussian error to solenoid strength
  - $B_{1,k} = B_{0,k}(1+d_{i,k})$
  - k=solenoids in linac
  - i=run through model
- Look at the relative difference in beta (Δβ/β)

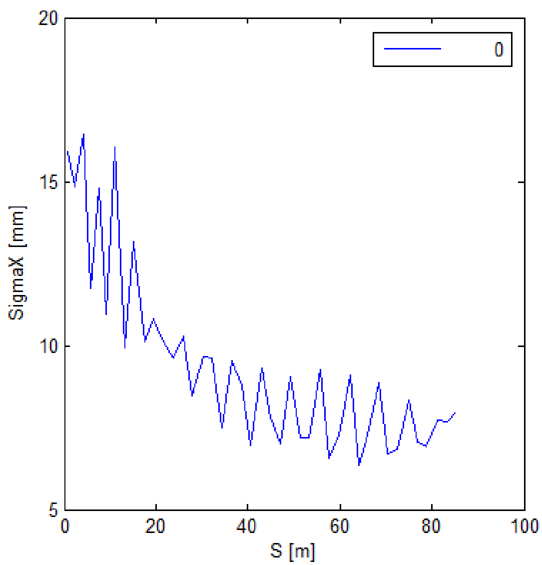


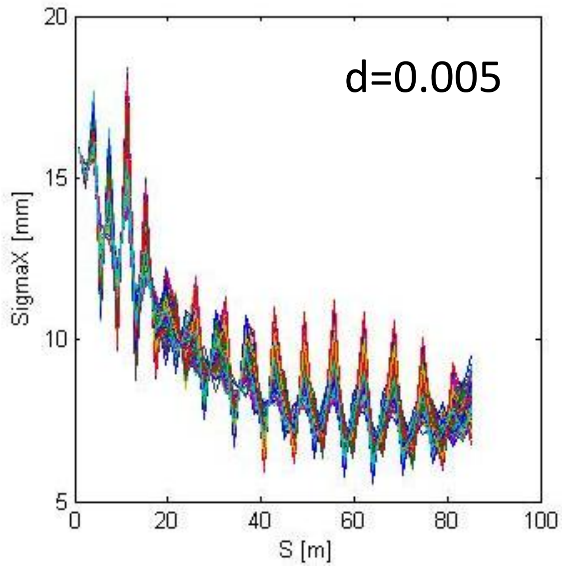
Solenoid Error Study

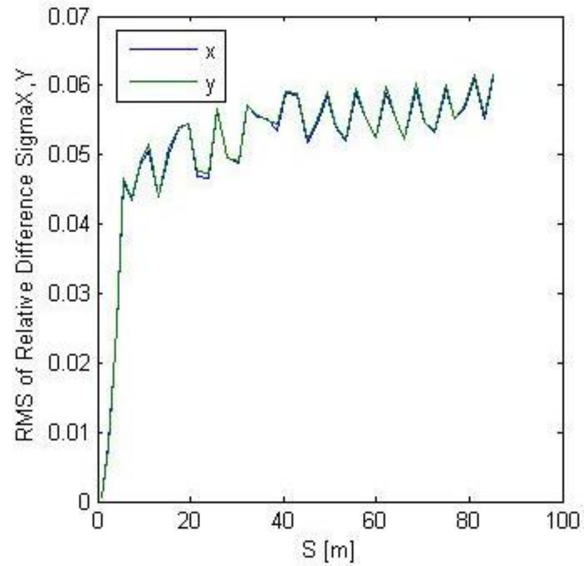


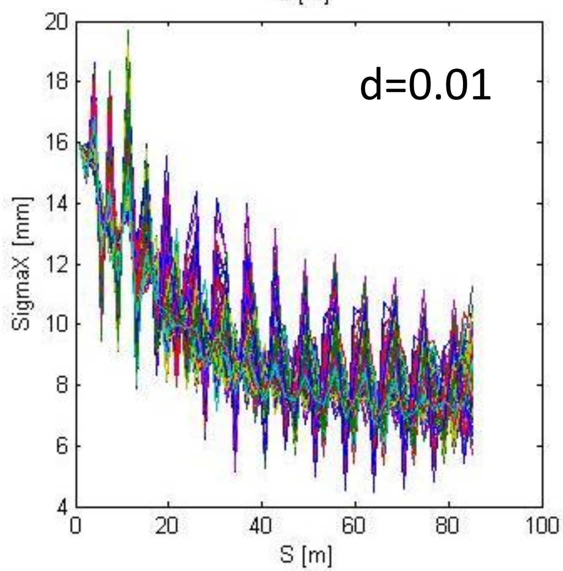
Solenoid Error Study

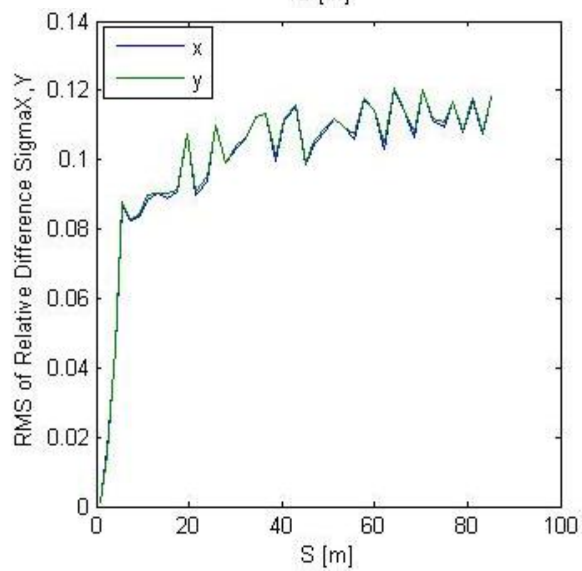
- XAL Model FRIB Seg1 Linac
- N=42 Solenoids
- Introduce Gaussian error to solenoid strength
  - $B_{1,k} = B_{0,k}(1+d_{i,k})$
  - k=solenoids in linac
  - i=run through model
- Look at sigma($\Delta\sigma_x/\sigma_x$)
- $\Delta\sigma_x/\sigma_x \sim d(\beta/f)(N/2)^{1/2}$



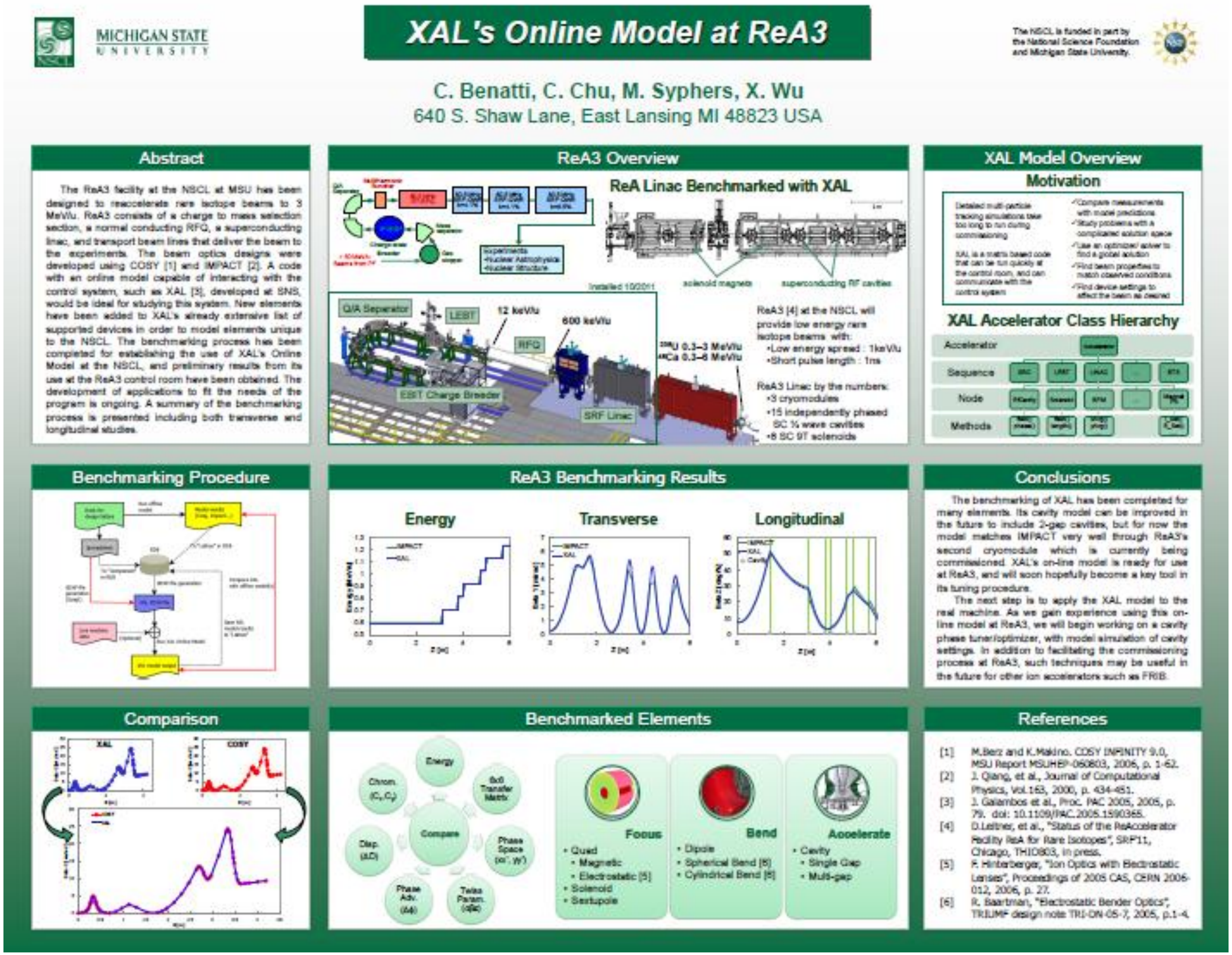




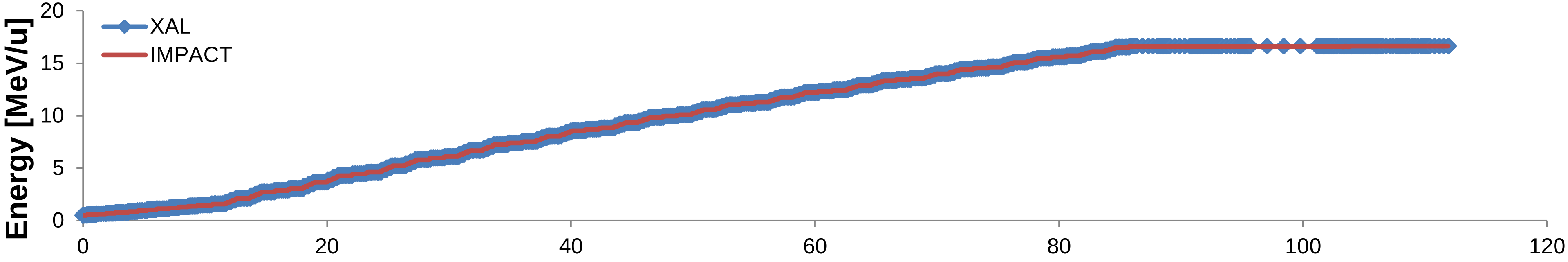


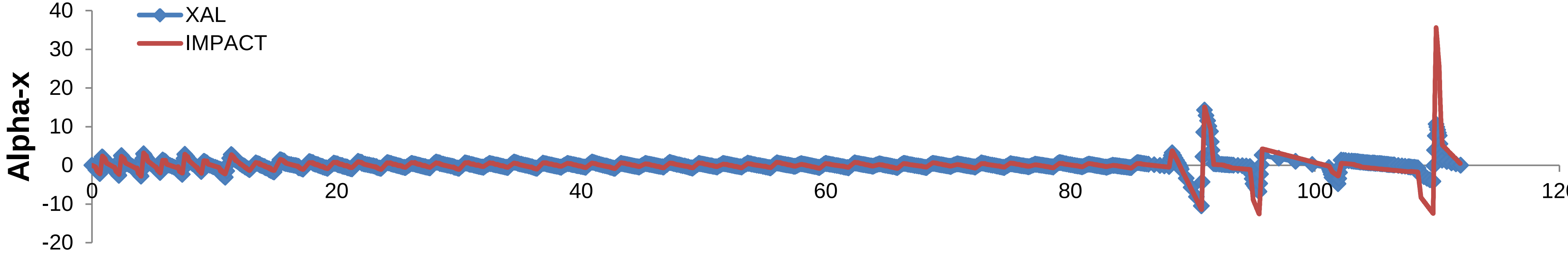


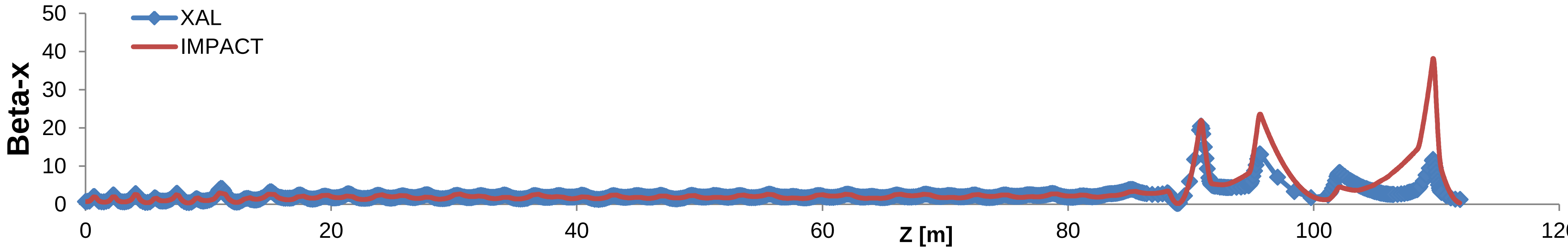
IPAC 2012 Poster



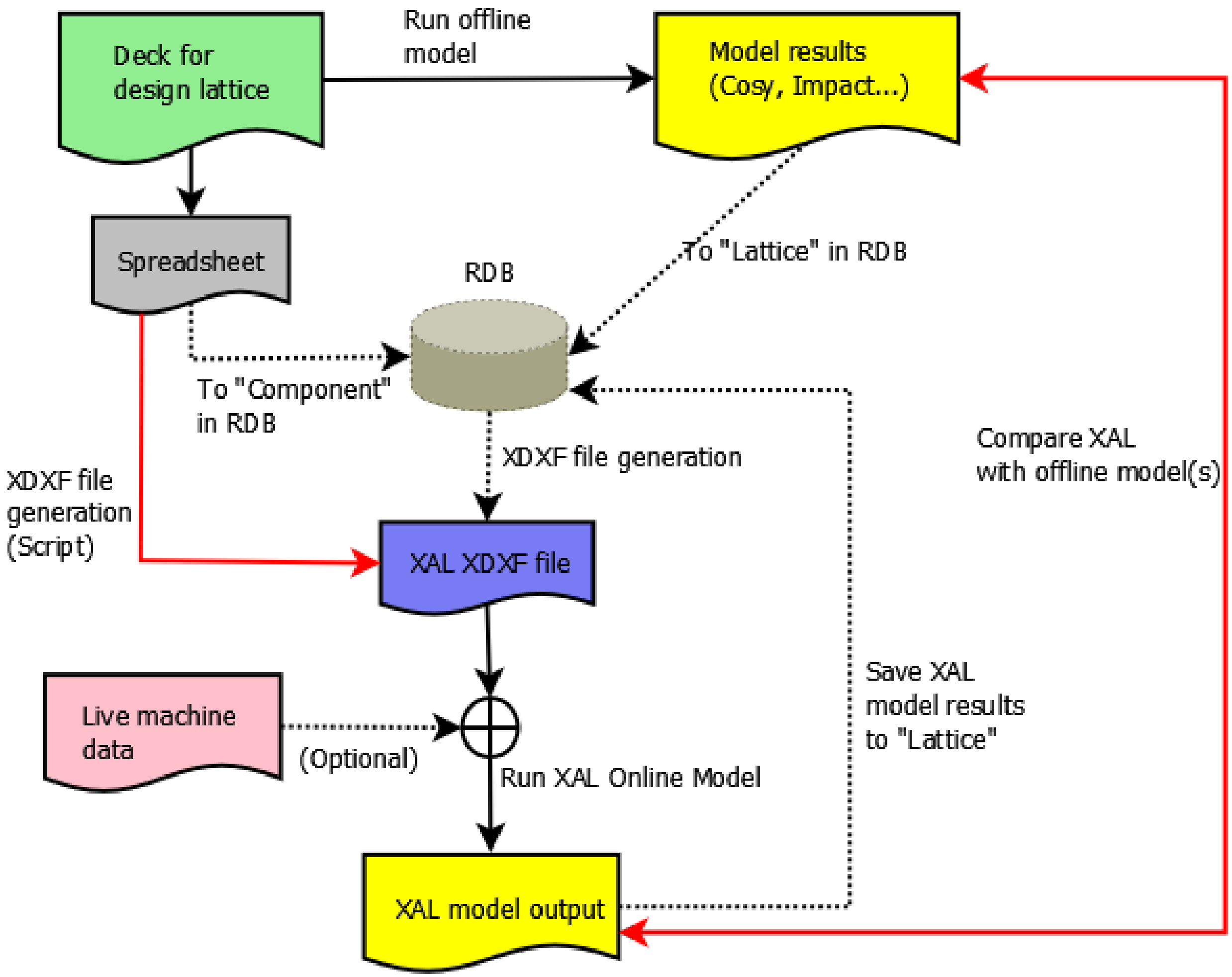
XAL-IMPACT FRIB Lattice: SEG1-Stripper







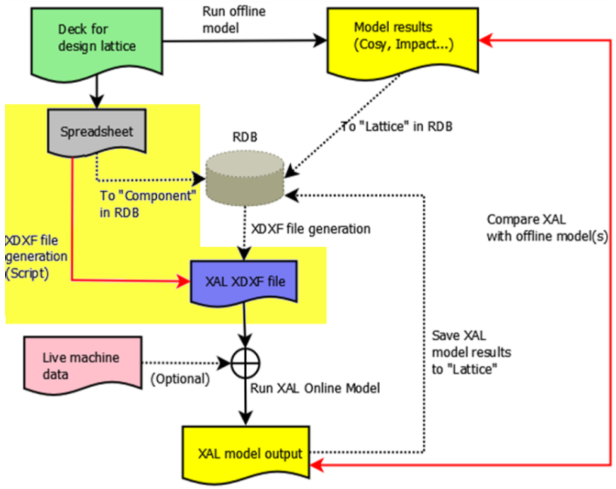
Model Benchmarking Process



XDXF File Generation

Excel File created from Lattice Spreadsheet File

Elements from LB Source to First BOB



System	Subsys.	Type	FRIB Name	NSCL Name	Length	Var. 1	Var. 2	Var. 3	Var. 4	FRIB Pos.	XAL Pos.
REA	BTS3	MARK	LBSOURCE_TO_RFQ_START	LBSOURCE_TO_RFQ_START	0					95.15597	0
REA	BTS3	DCH	REA_BTS2:DCH_D0954	LB003DH	0.1					95.37137	0.2154
REA	BTS3	DCV	REA_BTS2:DCV_D0954	LB003DV	0.1					95.37137	0.2154
REA	BTS3	QVE	REA_BTS3:QVE_D0954	LB004TA	0.1	-3.83279	0.03			95.37137	0.2154
REA	BTS3	QHE	REA_BTS3:QHE_D0955	LB004TB	0.1	5.511346	0.03			95.4924	0.336439
REA	BTS3	QVE	REA_BTS3:QVE_D0956	LB004TC	0.1	-3.40761	0.03			95.61344	0.457477
REA	BTS3	DCH	REA_BTS3:DCH_D0956	LB005DH	0.1					95.61344	0.457477
REA	BTS3	DCV	REA_BTS3:DCV_D0956	LB005DV	0.1					95.61344	0.457477
REA	BTS3	MARK	BOB LB	BOB LB	0					95.86079	0.704823

XDXF File


```
…
 <sequence id="LBSOURCE TO RFQ" len="4.84403392">
    <attributes>
      <sequence predecessors="null"/>
    </attributes>
    <node id="LBSOURCE_TO_RFQ_START" len="0" pid="LBSOURCE_TO_RFQ_START"
pos="0" s="95.15596608" type="MARK"/>
    <node id="REA_BTS2:DCH_D0954" len="0.1" pid="LB003DH" pos="0.2154"
s="95.37136608" type="DCH">
      <channelsuite name="magnetsuite">
        <channel handle="fieldReadH" settable="false" signal="LB003DH"/>
      </channelsuite>
    </node>
    <node id="REA_BTS2:DCV_D0954" len="0.1" pid="LB003DV" pos="0.2154"
s="95.37136608" type="DCV">
      <channelsuite name="magnetsuite">
        <channel handle="fieldReadV" settable="false" signal="LB003DV"/>
      </channelsuite>
    </node>
    <node id="REA_BTS3:QVE_D0954" len="0.1" pid="LB004TA" pos="0.2154"
s="95.37136608" type="QVE">
      <attributes>
        <magnet dfltMagFld="-3.8327921300" len="0.1" polarity="-1"/>
        <aperture x="0.03"/>
      </attributes>
      <channelsuite name="electrostaticsuite">
        <channel handle="voltageRead" settable="false" signal="LB004TA"/>
        <channel handle="voltageSetH" settable="true" signal="LB004TAHR"/>
        <channel handle="voltageSetV" settable="true" signal="LB004TAVT"/>
      </channelsuite>
    </node>
    <node id="REA_BTS3:QHE_D0955" len="0.1" pid="LB004TB" pos="0.3364385"
s="95.49240458" type="QHE">
      <attributes>
        <magnet dfltMagFld="5.5113455500" len="0.1" polarity="1"/>
        <aperture x="0.03"/>
      </attributes>
      <channelsuite name="electrostaticsuite">
        <channel handle="voltageRead" settable="false" signal="LB004TB"/>
        <channel handle="voltageSetH" settable="true" signal="LB004TBH"/>
        <channel handle="voltageSetV" settable="true" signal="LB004TBV"/>
      </channelsuite>
    </node>
    <node id="REA_BTS3:QVE_D0956" len="0.1" pid="LB004TC" pos="0.457477"
s="95.61344308" type="QVE">
      <attributes>
        <magnet dfltMagFld="-3.4076076400" len="0.1" polarity="-1"/>
        <aperture x="0.03"/>
      </attributes>
      <channelsuite name="electrostaticsuite">
        <channel handle="voltageRead" settable="false" signal="LB004TC"/>
        <channel handle="voltageSetH" settable="true" signal="LB004TCHR"/>
        <channel handle="voltageSetV" settable="true" signal="LB004TCVT"/>
      </channelsuite>
    </node>
    <node id="REA_BTS3:DCH_D0956" len="0.1" pid="LB005DH" pos="0.457477"
s="95.61344308" type="DCH">
      <channelsuite name="magnetsuite">
        <channel handle="fieldReadH" settable="false" signal="LB005DH"/>
        <channel handle="fieldSetH" settable="true" signal="LB005DHR"/>
      </channelsuite>
    </node>
    <node id="REA_BTS3:DCV_D0956" len="0.1" pid="LB005DV" pos="0.457477"
s="95.61344308" type="DCV">
      <channelsuite name="magnetsuite">
        <channel handle="fieldReadV" settable="false" signal="LB005DV"/>
        <channel handle="voltageSetV" settable="true" signal="LB005DVT"/>
      </channelsuite>
    </node>
    <node id="BOB LB" len="0" pid="BOB LB" pos="0.7048226" s="95.86078868"
type="MARK"/>
…
```

main.xal and model.params Files

main.xal

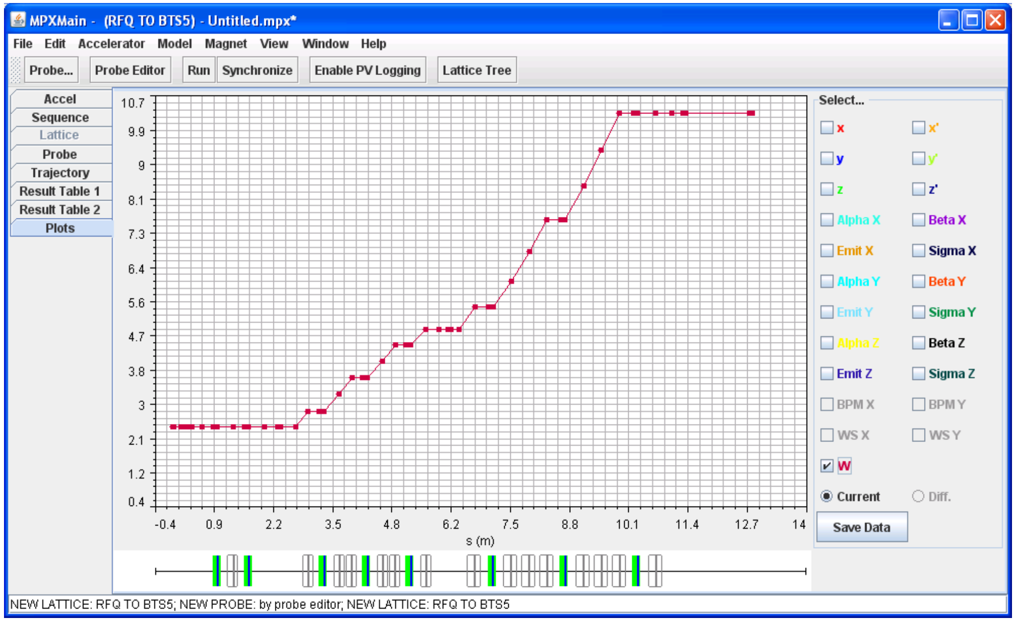
```
<?xml version = '1.0' encoding = 'UTF-8'?>
<!DOCTYPE sources SYSTEM "xdxf.dtd">
<sources>
 <deviceMapping_source name="deviceMapping" url="frib.impl"/>
 <optics_source name="optics" url="REA.xdxf"/>
 <timing_source name="timing" url="timing_pvs.tim"/>
 <tablegroup_source name="modelparams" url="REAmodel.params"/>
</sources>
```

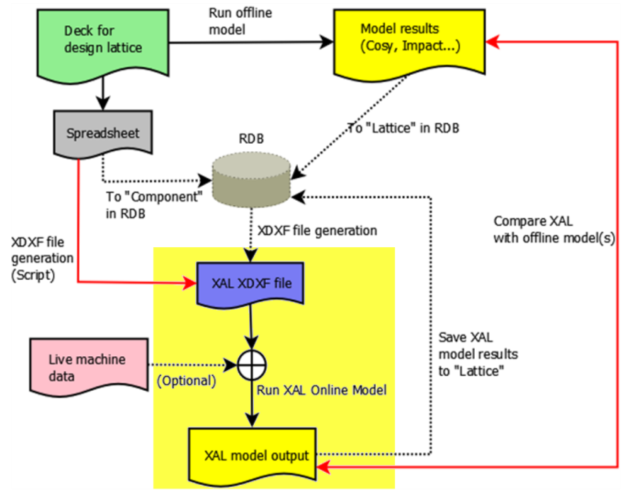
model.params

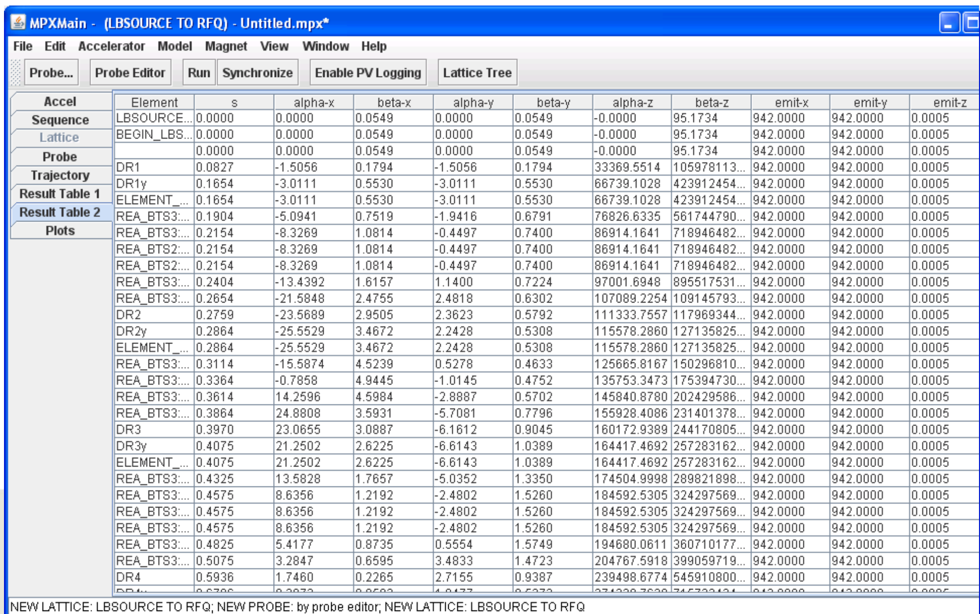
```
<?xml version = '1.0' encoding = 'UTF-8'?>
…
<table name="species">
…
   <record name="HELIUM" mass="3.72597728E9" charge="1"/>
</table>
…
<table name="twiss">
…
   <record name="LBSOURCE TO RFQ" coordinate="x" alpha="0" beta=".5492991115980524E-01" emittance=".1884000000000000E-03"/>
   <record name="LBSOURCE TO RFQ" coordinate="y" alpha="0" beta=".5492991115980524E-01" emittance=".1884000000000000E-03"/>
   <record name="LBSOURCE TO RFQ" coordinate="z" alpha="0" beta=".9609400000000001E-01" emittance=".1040647699127937E-10"/>
…
</table>
…
<table name="location">
…
   <record name="LBSOURCE TO RFQ" species="HELIUM" W="4.8047E4" s="0.0"/>
</table>
…
```

Units:
mass [eV/c^2]
alpha [rad]
beta [m/rad]
emit [π·m·rad]
W [eV] (total KE)

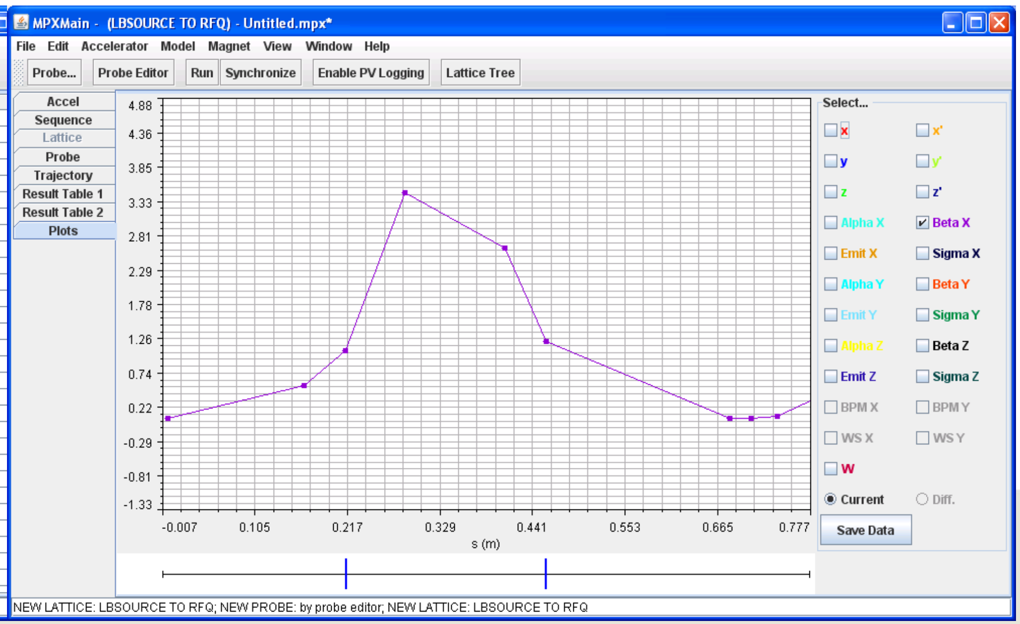
Run XAL Model--MPX





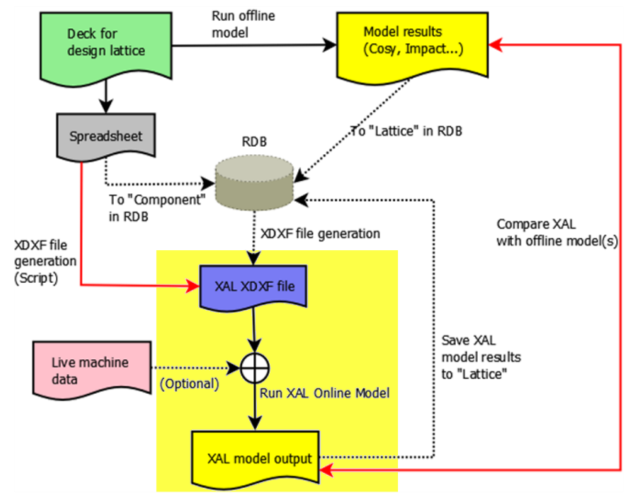


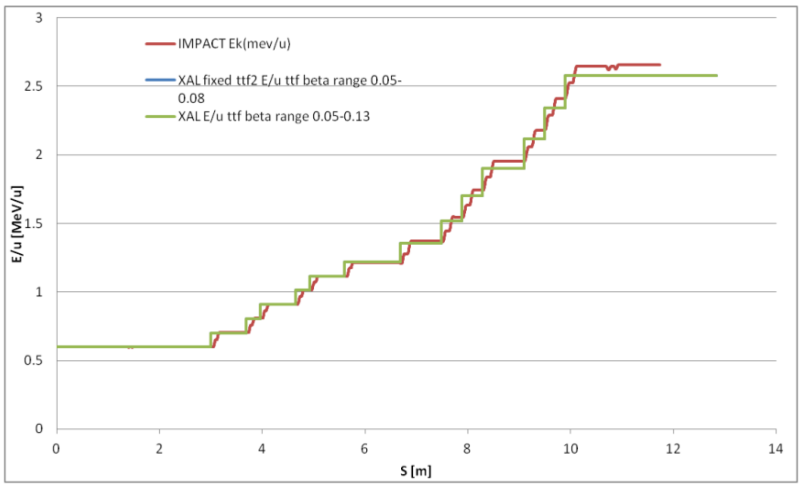
Element	s	alpha-x	beta-x	alpha-y	beta-y	alpha-z	beta-z	emit-x	emit-y	emit-z
LBSOURCE...	0.0000	0.0000	0.0549	0.0000	0.0549	-0.0000	95.1734	942.0000	942.0000	0.0005
BEGIN_LBS...	0.0000	0.0000	0.0549	0.0000	0.0549	-0.0000	95.1734	942.0000	942.0000	0.0005
	0.0000	0.0000	0.0549	0.0000	0.0549	-0.0000	95.1734	942.0000	942.0000	0.0005
DR1	0.0827	-1.5056	0.1794	-1.5056	0.1794	33369.5514	105978113...	942.0000	942.0000	0.0005
DR1y	0.1654	-3.0111	0.5530	-3.0111	0.5530	66739.1028	423912454...	942.0000	942.0000	0.0005
ELEMENT_...	0.1654	-3.0111	0.5530	-3.0111	0.5530	66739.1028	423912454...	942.0000	942.0000	0.0005
REA_BTS3:...	0.1904	-5.0941	0.7519	-1.9416	0.6791	76826.6335	561744790...	942.0000	942.0000	0.0005
REA_BTS3:...	0.2154	-8.3269	1.0814	-0.4497	0.7400	86914.1641	718946482...	942.0000	942.0000	0.0005
REA_BTS2:...	0.2154	-8.3269	1.0814	-0.4497	0.7400	86914.1641	718946482...	942.0000	942.0000	0.0005
REA_BTS2:...	0.2154	-8.3269	1.0814	-0.4497	0.7400	86914.1641	718946482...	942.0000	942.0000	0.0005
REA_BTS3:...	0.2404	-13.4392	1.6157	1.1400	0.7224	97001.6948	895517531...	942.0000	942.0000	0.0005
REA_BTS3:...	0.2654	-21.5848	2.4755	2.4818	0.6302	107089.2254	109145793...	942.0000	942.0000	0.0005
DR2	0.2759	-23.5689	2.9505	2.3623	0.5792	111333.7557	117969344...	942.0000	942.0000	0.0005
DR2y	0.2864	-25.5529	3.4672	2.2428	0.5308	115578.2860	127135825...	942.0000	942.0000	0.0005
ELEMENT_...	0.2864	-25.5529	3.4672	2.2428	0.5308	115578.2860	127135825...	942.0000	942.0000	0.0005
REA_BTS3:...	0.3114	-15.5874	4.5239	0.5278	0.4633	125665.8167	150296810...	942.0000	942.0000	0.0005
REA_BTS3:...	0.3364	-0.7858	4.9445	-1.0145	0.4752	135753.3473	175394730...	942.0000	942.0000	0.0005
REA_BTS3:...	0.3614	14.2596	4.5984	-2.8887	0.5702	145840.8780	202429586...	942.0000	942.0000	0.0005
REA_BTS3:...	0.3864	24.8808	3.5931	-5.7081	0.7796	155928.4086	231401378...	942.0000	942.0000	0.0005
DR3	0.3970	23.0655	3.0887	-6.1612	0.9045	160172.9389	244170805...	942.0000	942.0000	0.0005
DR3y	0.4075	21.2502	2.6225	-6.6143	1.0389	164417.4692	257283162...	942.0000	942.0000	0.0005
ELEMENT_...	0.4075	21.2502	2.6225	-6.6143	1.0389	164417.4692	257283162...	942.0000	942.0000	0.0005
REA_BTS3:...	0.4325	13.5828	1.7657	-5.0352	1.3350	174504.9998	289821898...	942.0000	942.0000	0.0005
REA_BTS3:...	0.4575	8.6356	1.2192	-2.4802	1.5260	184592.5305	324297569...	942.0000	942.0000	0.0005
REA_BTS3:...	0.4575	8.6356	1.2192	-2.4802	1.5260	184592.5305	324297569...	942.0000	942.0000	0.0005
REA_BTS3:...	0.4575	8.6356	1.2192	-2.4802	1.5260	184592.5305	324297569...	942.0000	942.0000	0.0005
REA_BTS3:...	0.4825	5.4177	0.8735	0.5554	1.5749	194680.0611	360710177...	942.0000	942.0000	0.0005
REA_BTS3:...	0.5075	3.2847	0.6595	3.4833	1.4723	204767.5918	399059719...	942.0000	942.0000	0.0005
DR4	0.5936	1.7460	0.2265	2.7155	0.9387	239498.6774	545910800...	942.0000	942.0000	0.0005



Run XAL Model--Matlab

```
:
%Run the XAL Online Model
%Read the accelerator
accl = XMLDataManager.acceleratorWithPath(acceleratorpath);
seq0 = accl.getSequence(sequencename);
%Model and Probe initializations
model = Scenario.newAndImprovedScenarioFor(seq0);
initProbe = ProbeFactory.getEnvelopeProbe(seq0, EnvTrackerAdapt(seq0));
model.resetProbe();
model.setProbe(initProbe);
model.setSynchronizationMode(Scenario.SYNC_MODE_DESIGN);
%Run model
model.run();
probe = model.getProbe();
traj = probe.getTrajectory();
:
```





Pos[m]	E[MeV]	PX	PA	EPSX	BETAX	ALPHAX
0	0.048047	0	0	0.000188	0.05493	0
0	0.048047	0	0	0.000188	0.05493	0
0	0.048047	0	0	0.000188	0.05493	0
0.0827	0.048047	0	0	0.000188	0.179439	-1.50555
0.1654	0.048047	0	0	0.000188	0.552967	-3.01111
0.1654	0.048047	0	0	0.000188	0.552967	-3.01111
0.1904	0.048047	0	0	0.000188	0.751938	-5.09412
0.2154	0.048047	0	0	0.000188	1.081402	-8.32692
0.2154	0.048047	0	0	0.000188	1.081402	-8.32692
0.2154	0.048047	0	0	0.000188	1.081402	-8.32692
0.2404	0.048047	0	0	0.000188	1.615724	-13.4392
0.2654	0.048047	0	0	0.000188	2.475507	-21.5848
0.275919	0.048047	0	0	0.000188	2.950489	-23.5689
0.286439	0.048047	0	0	0.000188	3.467213	-25.5529
0.286439	0.048047	0	0	0.000188	3.467213	-25.5529

...

:

Run COSY Model

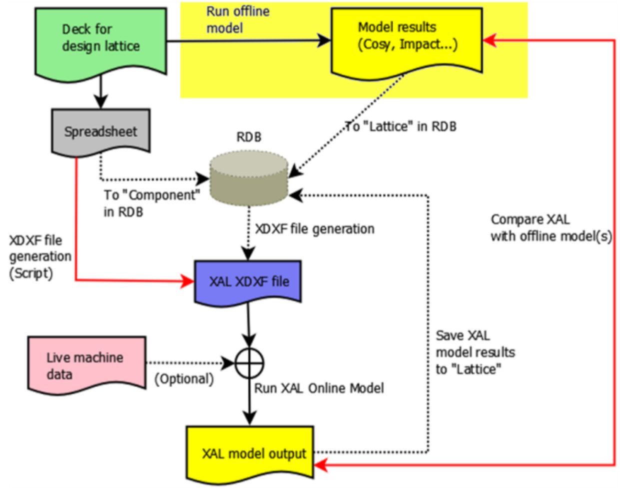
:

```
 PHASE SPACE 6
SPOS:   .5054000000000000

----  PHASE SPACE PARAMETERS----
  E0[MeV]=   .4804700000000000E-01
  CHIM[Tm]=   .6311737672167199E-01
  V0/CLIGHT=   .5078364094136823E-02
  A0[amu]=   4.000000000000000
  E0[MeV]=   .4804700000000000E-01
  Z0[e units]=   1.000000000000000
--  X-A                       --
 EPSXN,   .9567761329388078E-06
 EPSX,   .1883999999999998E-03
 BETAX,   .7110814434861047  ALPHAX,   3.163199329699511
 PX =  .1157E-01 PA =  .5400E-01
   R12 = -.9535E+00
--  Y-B                       --
 EPSYN,   .9567761329388087E-06
 EPSY,   .1884000000000000E-03
 BETAY,   1.478984457766634  ALPHAY,   3.452747755090183
 PY =  .1669E-01 PB =  .4057E-01
   R34 = -.9605E+00
--  L-D                       --
 EPSLN,   .5284856057979282E-13
 EPSL,   .1040647699127937E-10
 BETAL,   .2622220702466099  ALPHAL,  -1.314841412728922
 PT =  .1652E-05 PD =  .1041E-04
   R56 =  .7960E+00
```

:

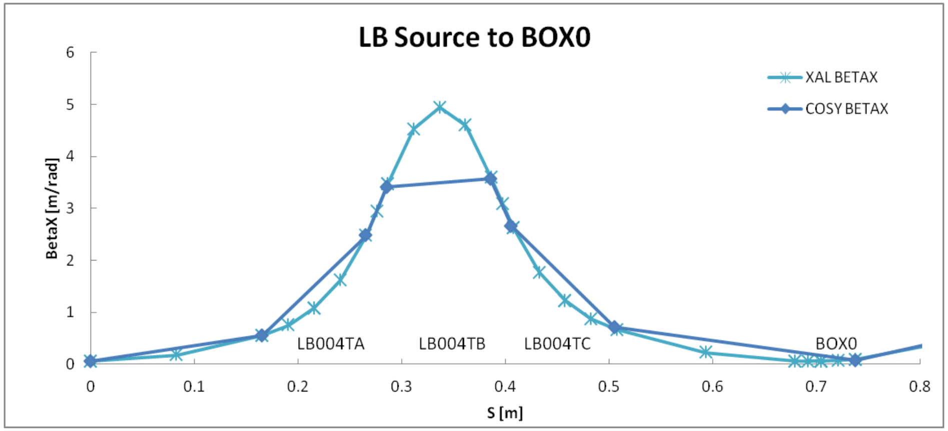
- Create COSY ".fox" file
- Run COSY (Cosy8a)
- Matlab script converts COSY output to Excel file



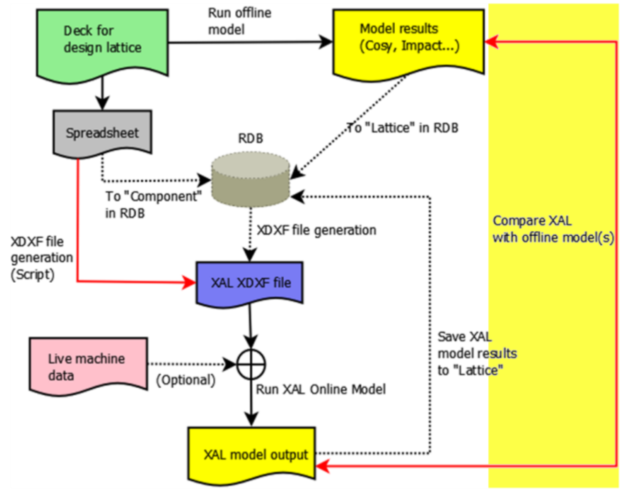
Pos[m]	E[MeV]	PX	PA	EPSX	BETAX	ALPHAX	PY	PB
0	0.048047	0.003217	0.05856	0.000188	0.05493	0	0.003217	0.05856
0.1654	0.048047	0.01021	0.05856	0.000188	0.552967	-3.01111	0.01021	0.05856
0.2654	0.048047	0.0216	0.1885	0.000188	2.475516	-21.585	0.0109	0.04626
0.2854	0.048047	0.02536	0.1885	0.000188	3.414359	-25.3572	0.01004	0.04626
0.3854	0.048047	0.02593	0.1783	0.000188	3.569975	24.52617	0.01219	0.09062
0.4054	0.048047	0.02237	0.1783	0.000188	2.656439	21.15061	0.01398	0.09062
0.5054	0.048047	0.01157	0.054	0.000188	0.711081	3.163199	0.01669	0.04057
0.737743	0.048047	0.003802	0.054	0.000188	0.076718	-0.43291	0.008076	0.04057
0.970086	0.048047	0.01448	0.054	0.000188	1.113415	-4.02902	0.005432	0.04057
1.297335	0.048047	0.0167	0.04253	0.000188	1.480761	3.635297	0.0106	0.01795
1.972168	0.048047	0.01335	0.04253	0.000188	0.946182	-2.84313	0.01408	0.01392
2.304421	0.048047	0.02709	0.04253	0.000188	3.895229	-6.03278	0.01598	0.01392
2.636674	0.048047	0.04109	0.04253	0.000188	8.963817	-9.22242	0.01886	0.01392
2.786674	0.048047	0.04012	0.05511	0.000188	8.543111	11.6932	0.02384	0.05907
2.816674	0.048047	0.03847	0.05511	0.000188	7.856029	11.20954	0.0256	0.05907
2.966674	0.048047	0.03844	0.05474	0.000188	7.844964	-11.1252	0.02832	0.02454
3.298724	0.048047	0.05657	0.05474	0.000188	16.98681	-16.4063	0.0206	0.02454
3.448724	0.048047	0.04912	0.1494	0.000188	12.80742	38.94095	0.02286	0.05492
3.478724	0.048047	0.04464	0.1494	0.000188	10.57759	35.38661	0.02449	0.05492
3.628724	0.048047	0.03428	0.006918	0.000188	6.236717	-0.76447	0.02473	0.05187
3.808224	0.048047	0.03505	0.006918	0.000188	6.519348	-0.81007	0.01558	0.05187

...

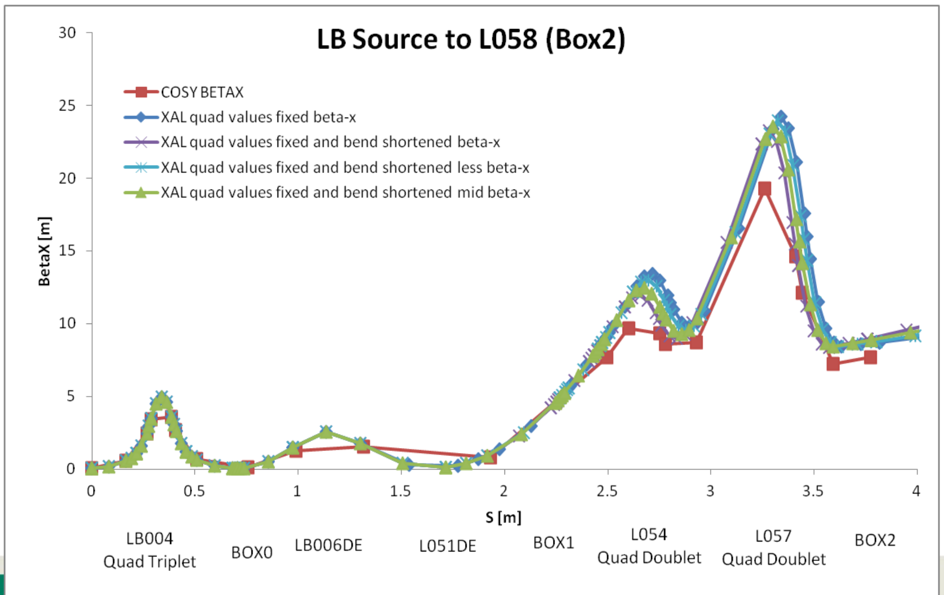
Comparison



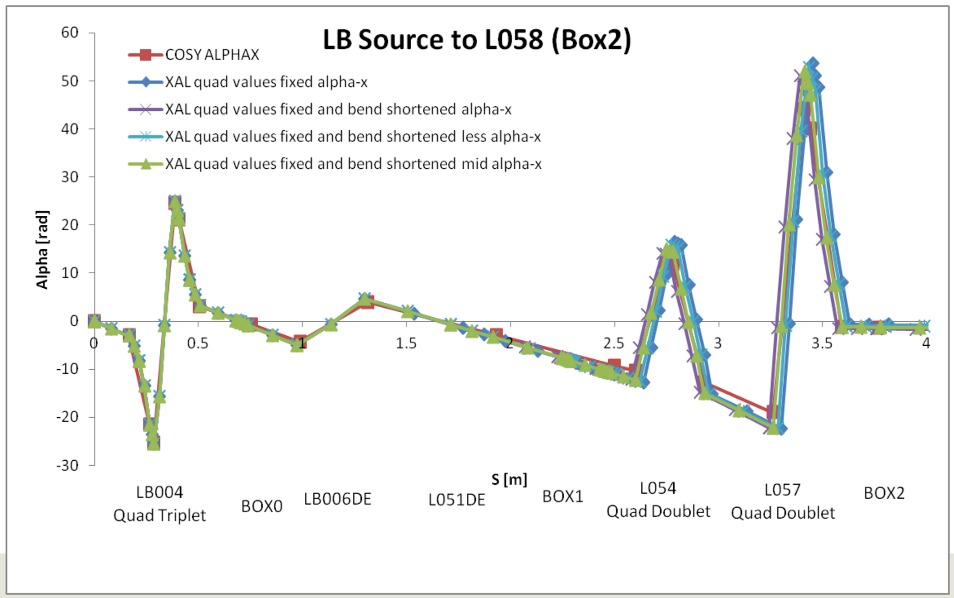
LB Source to BOX0
XAL BETAX
COSY BETAX
BetaX [m/rad]
S [m]
LB004TA
LB004TB
LB004TC
BOX0



Deck for design lattice
Run offline model
Model results (Cosy, Impact...)
Spreadsheet
RDB
To "Lattice" in RDB
To "Component" in RDB
XDXF file generation
XDXF file generation (Script)
XAL XDXF file
Compare XAL with offline model(s)
Live machine data
(Optional)
Run XAL Online Model
Save XAL model results to "Lattice"
XAL model output

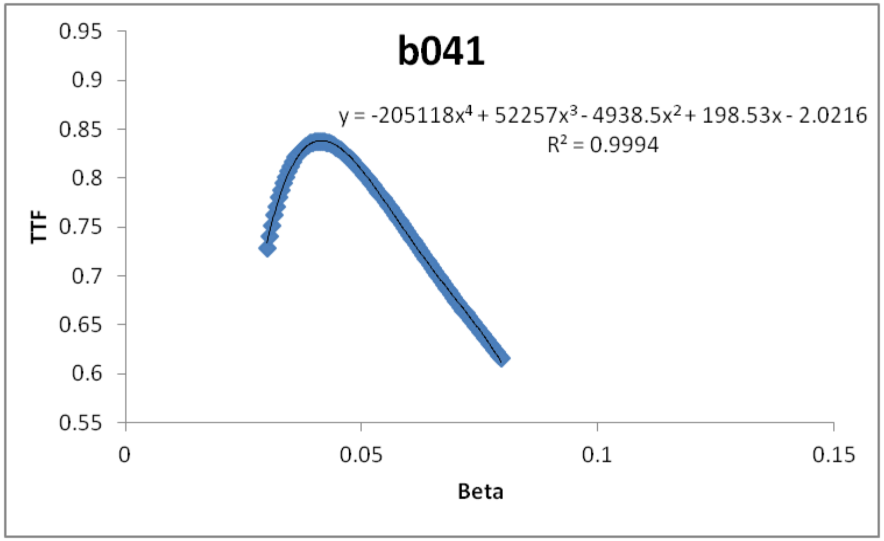


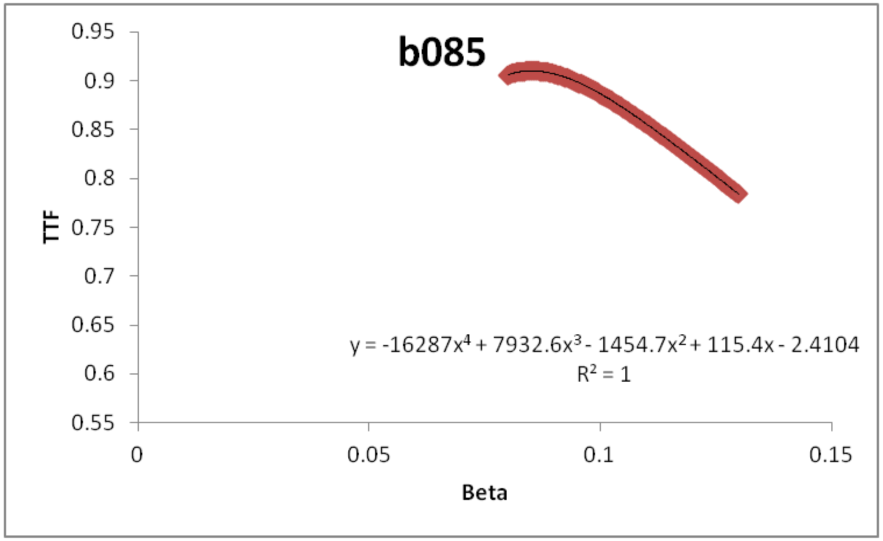
LB Source to L058 (Box2)
COSY BETAX
XAL quad values fixed beta-x
XAL quad values fixed and bend shortened beta-x
XAL quad values fixed and bend shortened less beta-x
XAL quad values fixed and bend shortened mid beta-x
BetaX [m]
S [m]
LB004 Quad Triplet
BOX0
LB006DE
L051DE
BOX1
L054 Quad Doublet
L057 Quad Doublet
BOX2

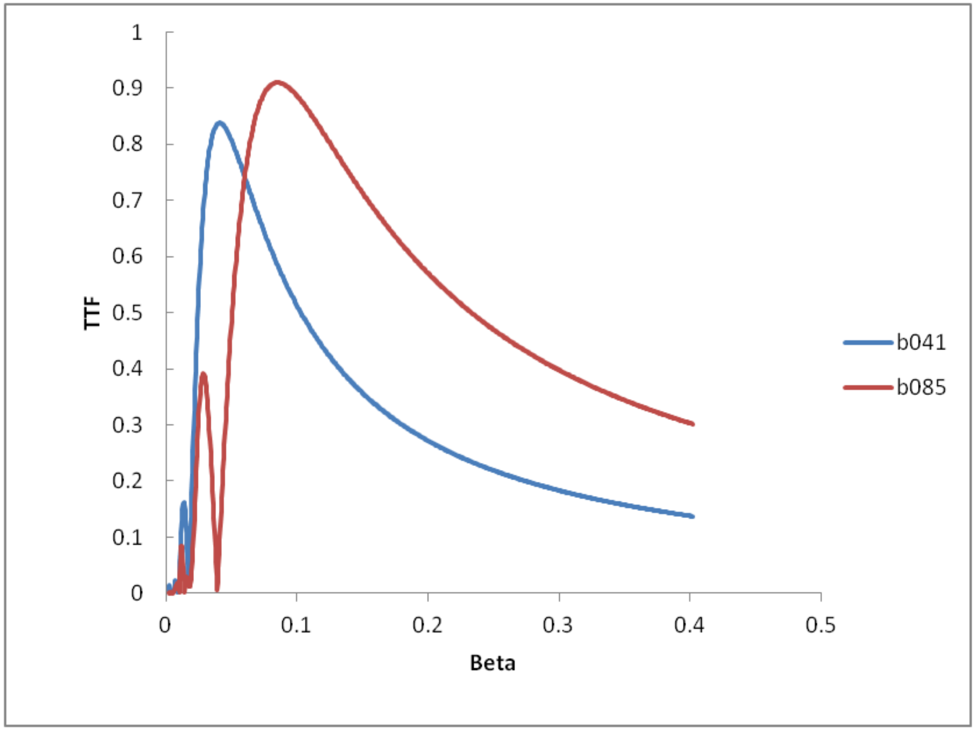


LB Source to L058 (Box2)
COSY ALPHAX
XAL quad values fixed alpha-x
XAL quad values fixed and bend shortened alpha-x
XAL quad values fixed and bend shortened less alpha-x
XAL quad values fixed and bend shortened mid alpha-x
Alpha [rad]
S [m]
LB004 Quad Triplet
BOX0
LB006DE
L051DE
BOX1
L054 Quad Doublet
L057 Quad Doublet
BOX2

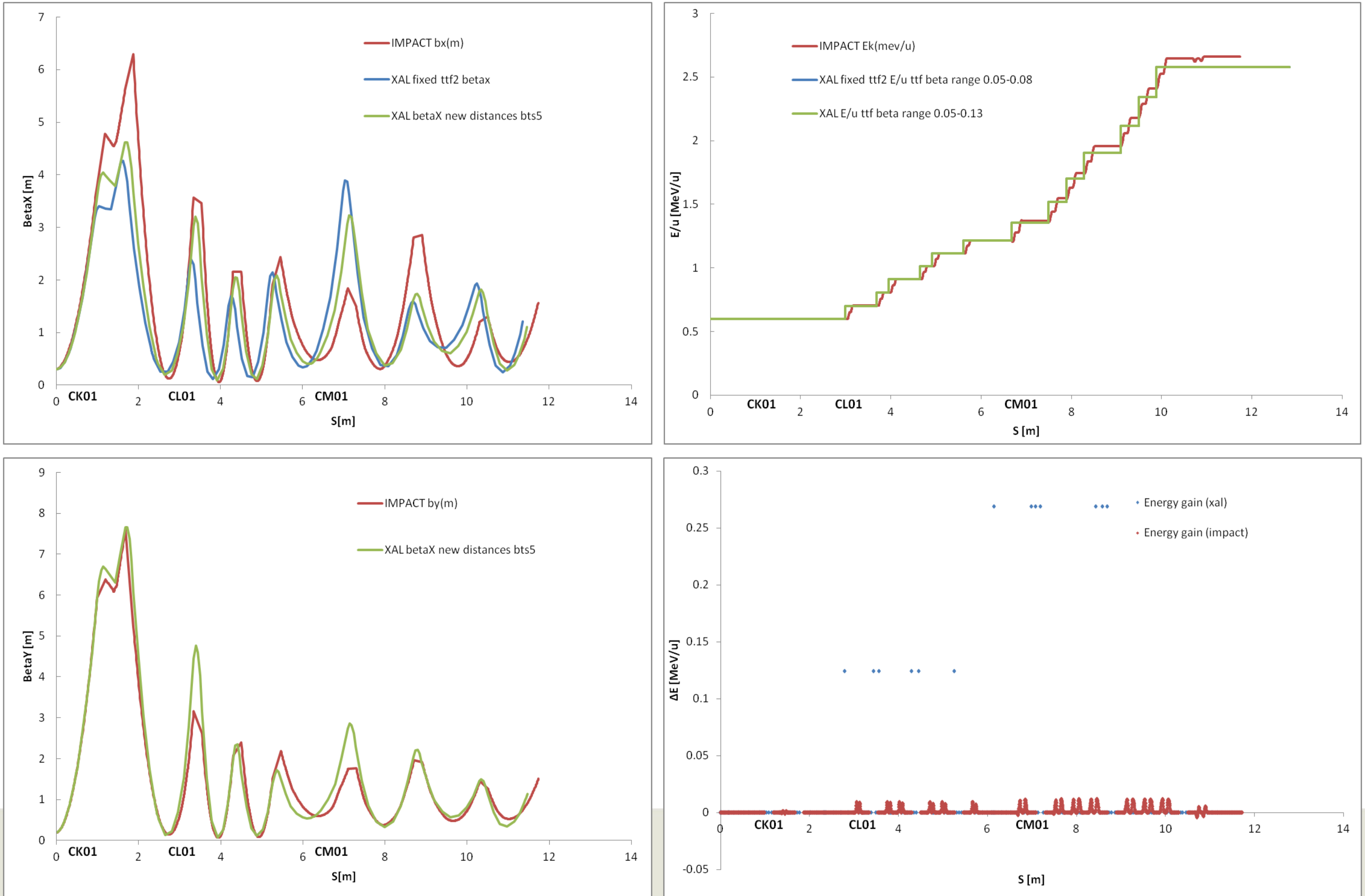
TTF Fit for ReA Cavities





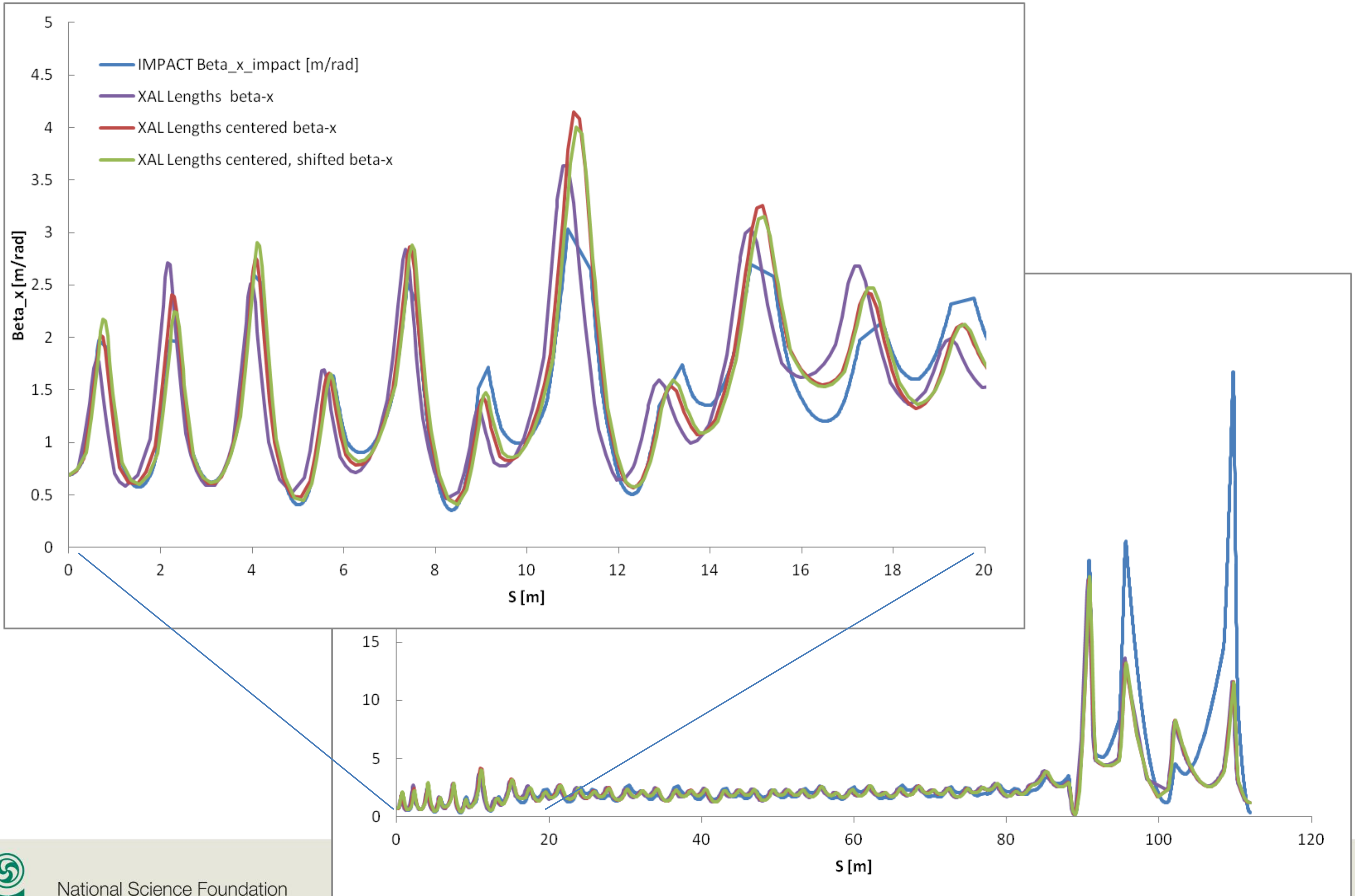


RFQ to BTS5



NSCL

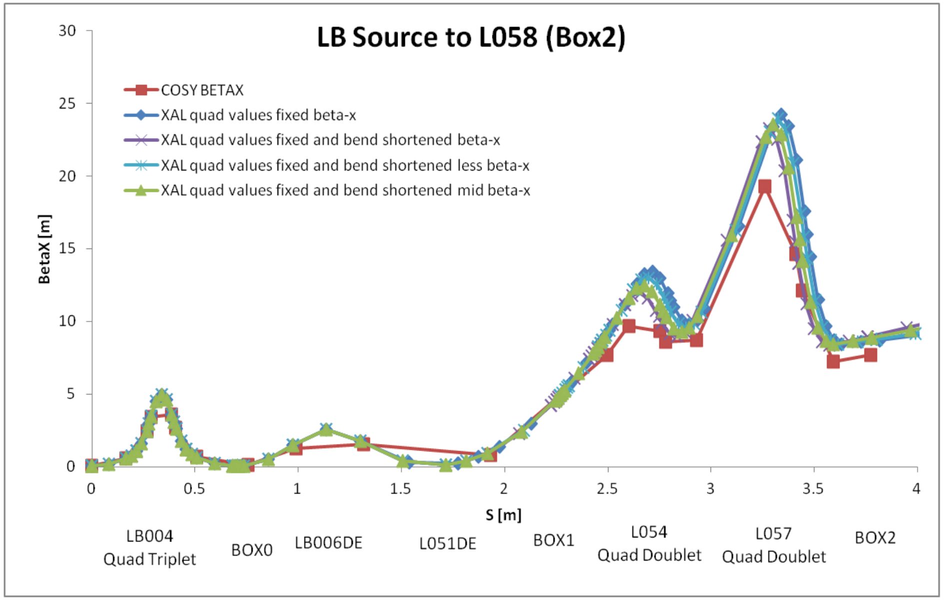
FRIB Seg1 to Stripper



Summary

Issues:
- Transverse Twiss parameters
    - Before and after RFQ
    - FRIB Seg1 at quads
- Energy gain b085 cavities
- Sph/Cyl bend new device vs. new type variable added to XAL

Resolved:
- Transfer matrices established
    - Cylindrical bend
    - Spherical bend
- Longitudinal coordinate units
- Rotation matrix b/w COSY, WAL
- MPX units, factor of 5 in emit.



Bquad

Bquad—Horizontal
Difference

Pos[m]	E[MeV]	EPSX	BETAX	ALPHAX	EPSY	BETAY	ALPHAY
0	0	0	0	0	0	0	0
0.1	0	0	0.000682	-0.00618	0	-0.00092	0.009894

Percent Difference

0	0	0	-1.5E-16	-3.2E-16	-5.3E-16	3.33E-16	8.54E-16
0.1	0	2.96E-16	0.000943	-0.00654	-3.6E-16	-0.00094	-0.00894

Bquad—Vertical
Difference

Pos[m]	E[MeV]	EPSX	BETAX	ALPHAX	EPSY	BETAY	ALPHAY
0	0	0	0	0	0	0	0
0.1	0	1.02E-14	-0.00092	0.011077	0	0.000656	-0.00469

Percent Difference

0	0	0	-1.5E-16	-3.2E-16	-5.3E-16	3.33E-16	8.54E-16
0.1	0	3.4E-15	-0.00082	-0.0033	1.07E-15	0.001121	-0.00187

Bquad—Horizontal
Rmat
Percent Difference

0.050921	0.015862	0	0
-0.37987	0.050921	0	0
0	0	-0.04345	-0.01537
0	0	-0.41123	-0.04345

Bquad—Vertical
Rmat
Percent Difference

-0.04345	-0.01537	0	0
-0.41123	-0.04345	0	0
0	0	0.050921	0.015862
0	0	-0.37987	0.050921