Open XAL Acceleration Model Upgrade

Architecture, Verification, Modeling Issues

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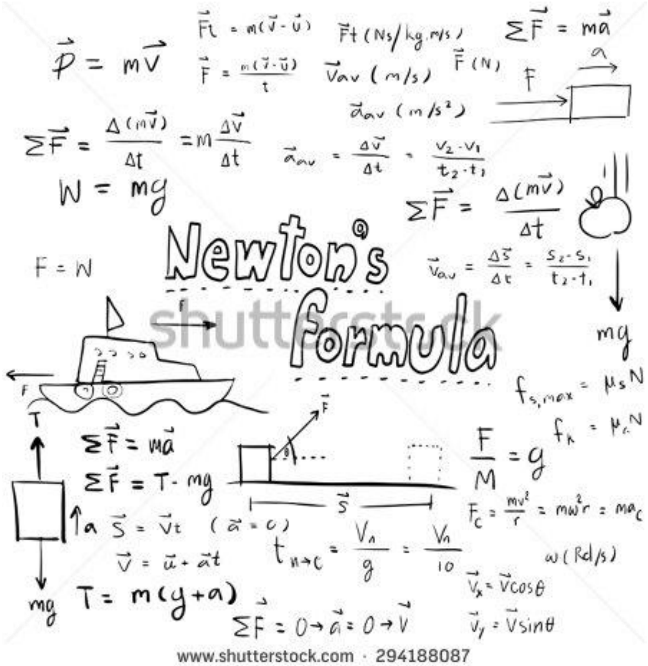
Objectives

- Create generalized (thin-lens) RF acceleration model
  - Better predict SNS linac (SCL) ?
  - No field restrictions

- Retire the currently used XAL version
  - Repair/replace earlier “quick fixes”
  - Hierarchical acceleration model.



full field = no assumptions

Tasks

- Needed RF cavity software model (hierarchical)
  - **Need (major) modification to lattice generator** to support nested modeling elements
  - Probes must carry their own phase information
  - Algorithms modified to support phase
  - RF cavity container/manager class
    - Cell indices, modes, amplitudes, phases, etc.
  - New RF gap class

generated a cascade of upgrades to Open XAL

- Need tools to compute new spectral quantities

- Architectural support for new data
  - storage, retrieval, encapsulation

Current Status

- All the aforementioned issued have been addressed

- The new RF acceleration model has been analyzed and **documented** extensively
  - C.K. Allen "A Thin-Lens Model for Charged-Particle RF Accelerating Gaps," ORNL Report #TM-2017/395 (July, 2017).

- A tool form computing spectral quantities from fields maps has been written, *TTF Workshop*
  - C.K. Allen, James Ghawaly, with help from A. Shishlo

- Currently verifying and debugging new acceleration model

Software Architecture

RF Cavity Design

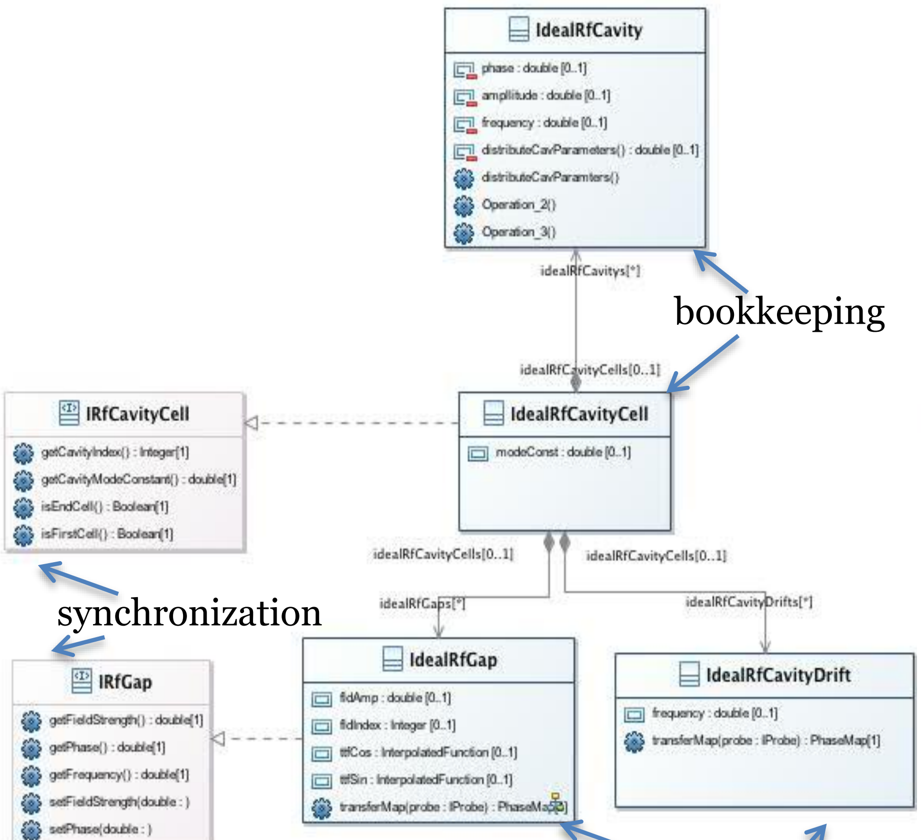
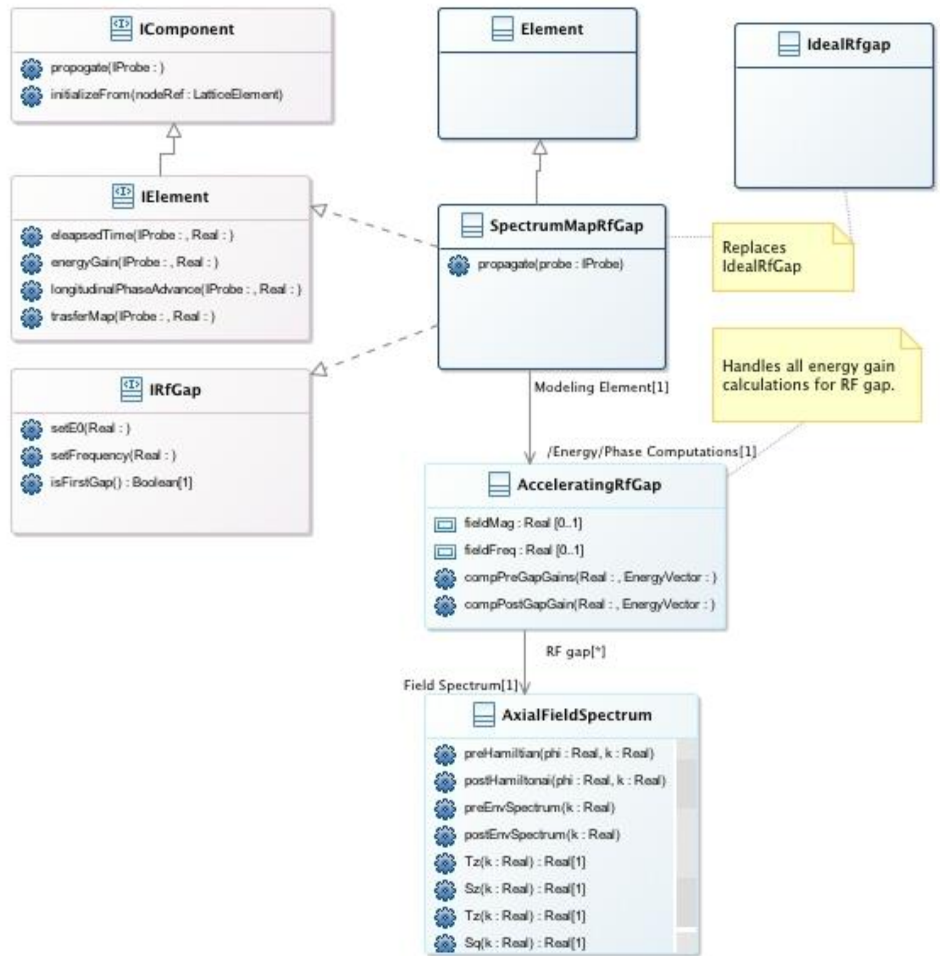


Figure 31: RF cavity software design

RF Gap Design



Spectral Data Locations

- optics [optics allenck.ttfs-for-new-accelmod.ver1]
  - design
    - alignment.xdxf
    - corrector-polarities.xdxf
    - device-mods.xdxf
    - hardware_status.xdxf
    - main.xal
    - rfgap_ttfs.xdxf
    - sns-model.impl
    - sns-model.params
    - sns-smf.impl
    - sns.xdxf
    - snsTTFpolys.xdxf
    - timing_pvs.tim
    - ttfExtras.xdxf
    - xal.mod.lattice.dtd
    - xal.mod.probe.dtd
    - xdxf.dtd
  - fieldmap
    - linacparams
      - betas.xml
      - SnsLinacStructureParameters.xml
    - superfish
      - ccl
      - dtl
      - mebt
      - scl
      - .DS_Store
    - ttf
      - ccl
      - dtl
      - mebt
      - scl
      - .DS_Store
      - README-PolyDomaindDesc.txt

Superfish data

processed spectral data

main.xal

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Verification of Upgrades

- Currently verifying operation of acceleration model
  - particular relevance to the SCL
  - comparing results of online model to analytic model with direct numerical integration using Mathematica

verification – does the model compute what it is supposed to compute?

validation – does the model compute the answer? (reality)

Verification: Do they compute the same thing?

- <u>Computer Model</u>
  - Augmented Panofsky Eq.
  - Discrete, coupled, transcendental equations

$$\phi_0 = \phi_0^- + Im\left[K^- \frac{\partial}{\partial ik} H^-(\phi_0 + \bar{k}^-)\right],$$
$$W_0 = W^- - Im\left[\frac{\partial}{\partial \phi} H^-(\phi_0 + \bar{k}^-)\right],$$

where

$$H^-(\phi + ik) \triangleq qV_0 e^{-i\phi}\mathcal{E}^-(ik),$$
$$H^+(\phi + ik) \triangleq qV_0 e^{-i\phi}\mathcal{E}^+(ik).$$

Laplace transform spectrum of $E_z(z)$
(defines the gap)

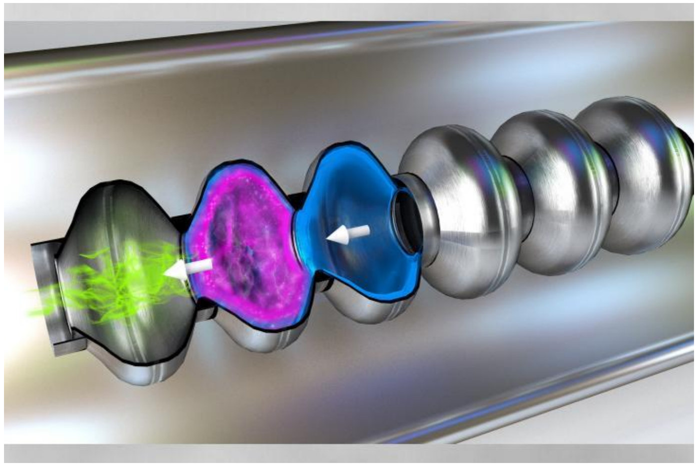
- <u>Analytic Model</u>
  - Coupled 1st order ODEs

$$\phi'(z) = k[W(z)],$$
$$W'(z) = qE_z(z)\cos\phi(z),$$

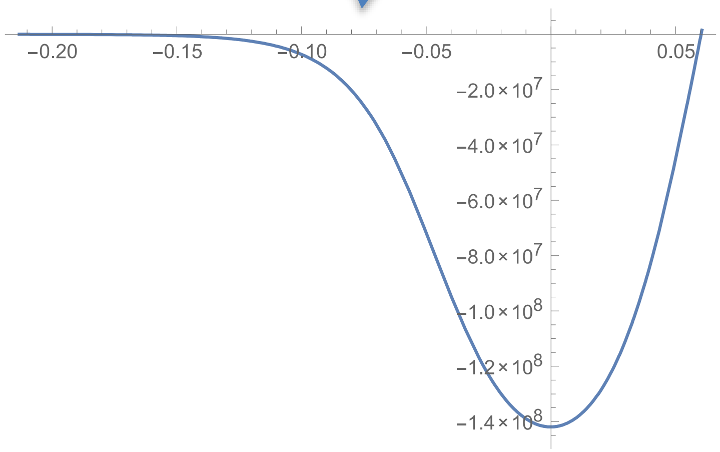
Longitudinal electric field
(defines the gap)

Single-Gap:

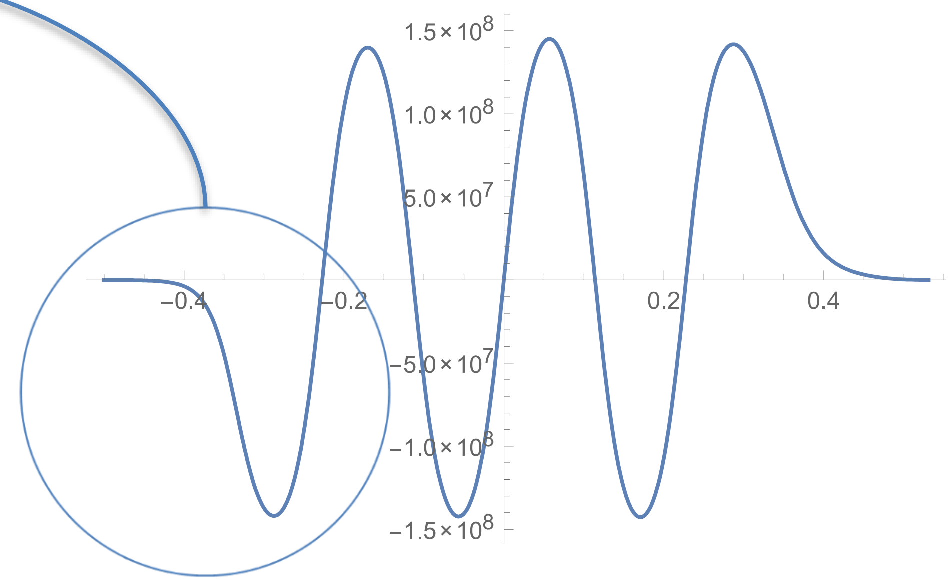
Test Example Medium-β SCL:Cav01a:Rg01



- Single gap:
  SCL:Cav01a:Rg01

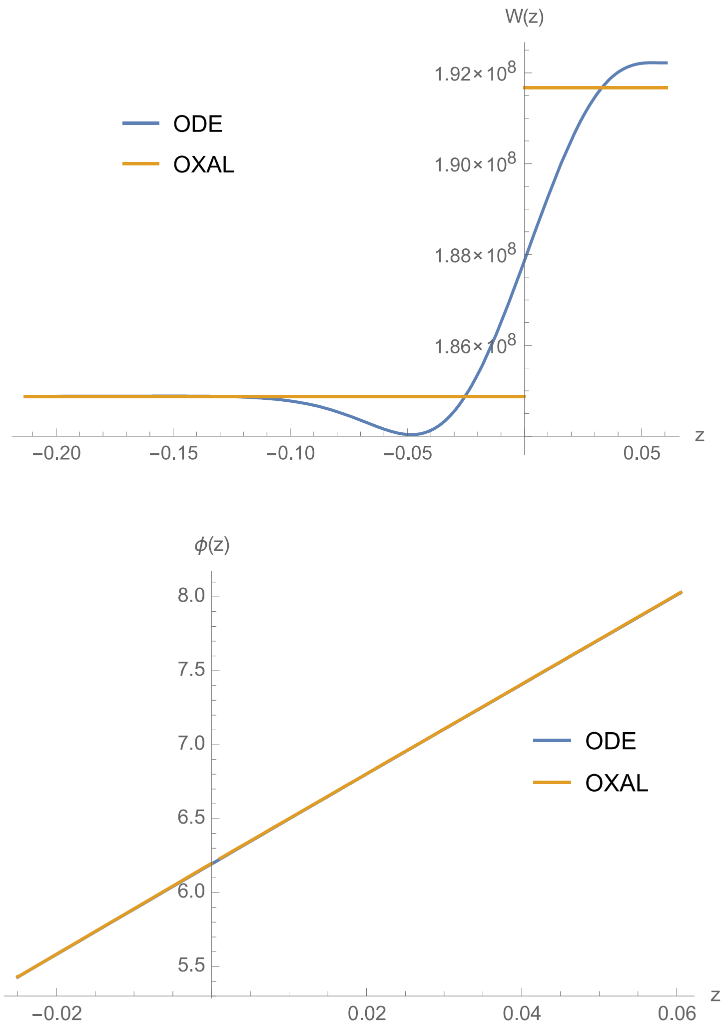


- Superfish Field Map:
  SCL:Cav01a:Rg01- Rg06

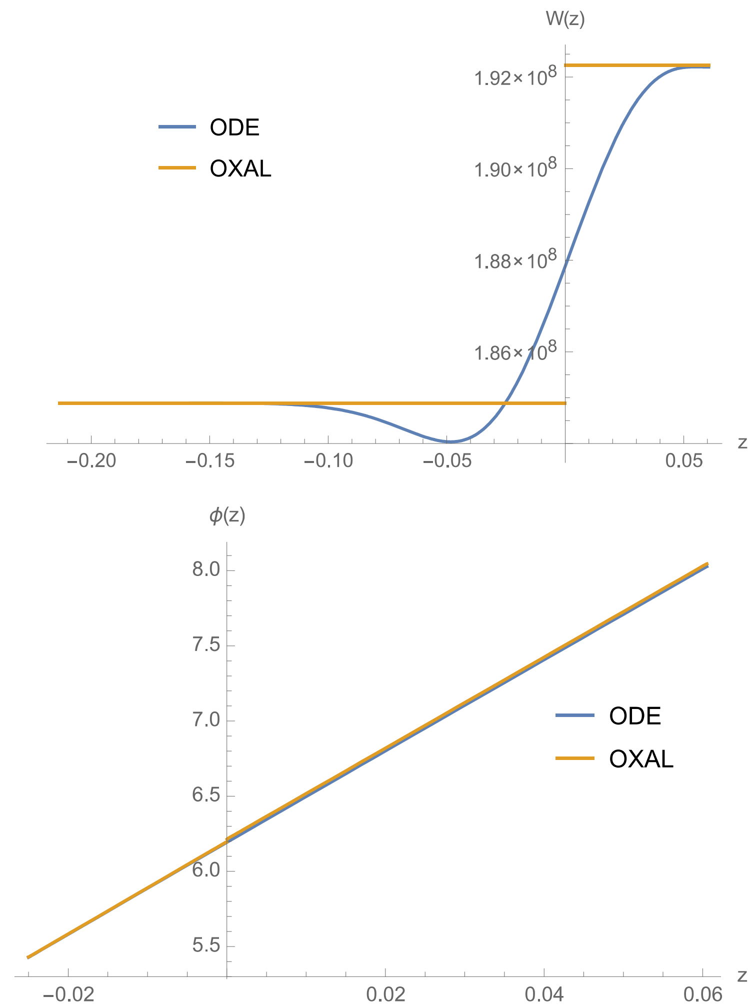


Single-Gap: Simulation SCL:Cav01a:Rg01
Online Model vs. Analytic – Old and New

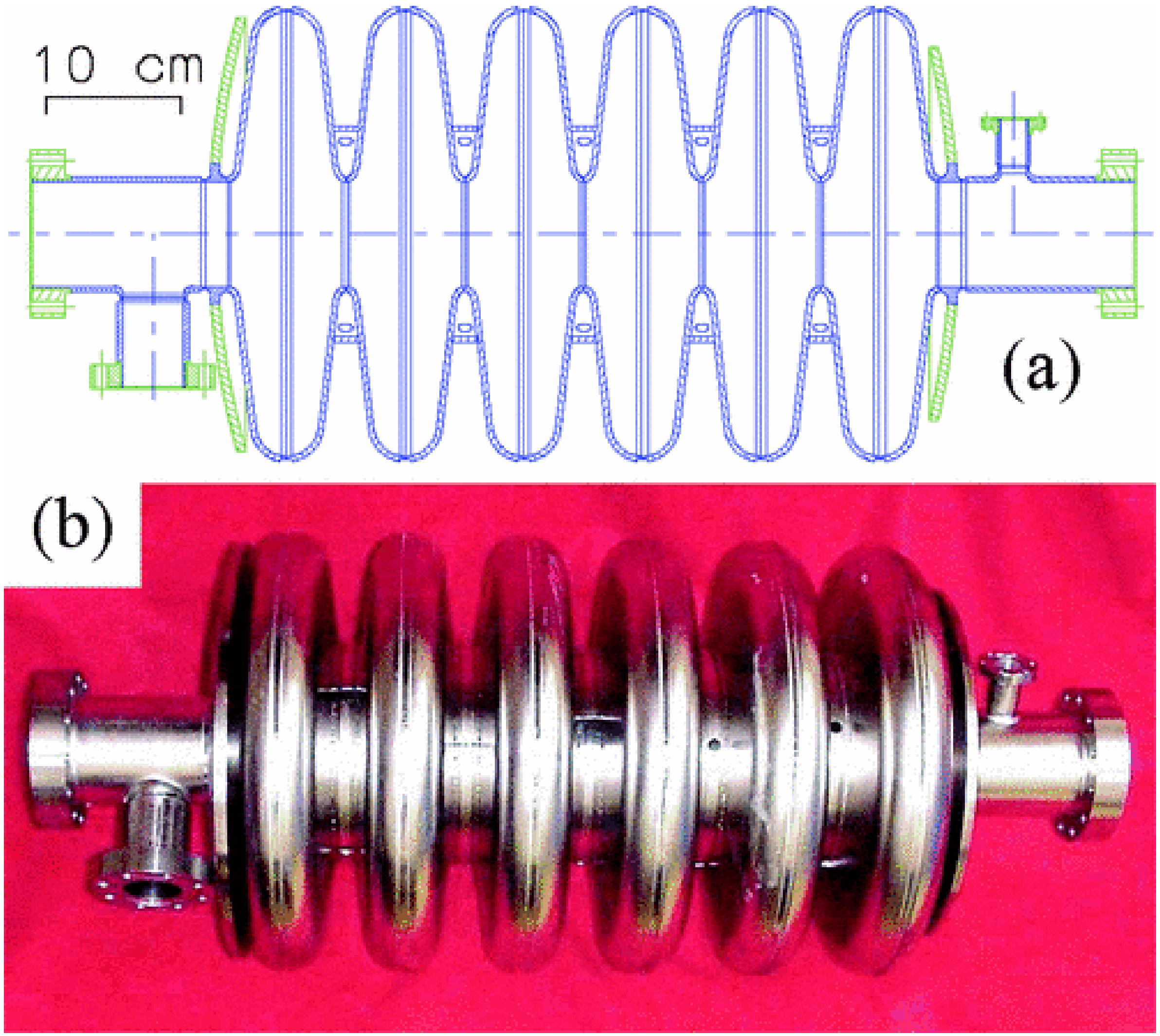
- IdealRfGap (old)

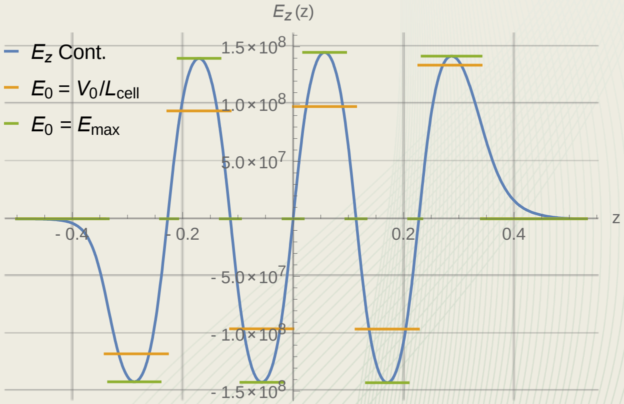


- SpectrumMapRfGap (new)



Cavity Analysis

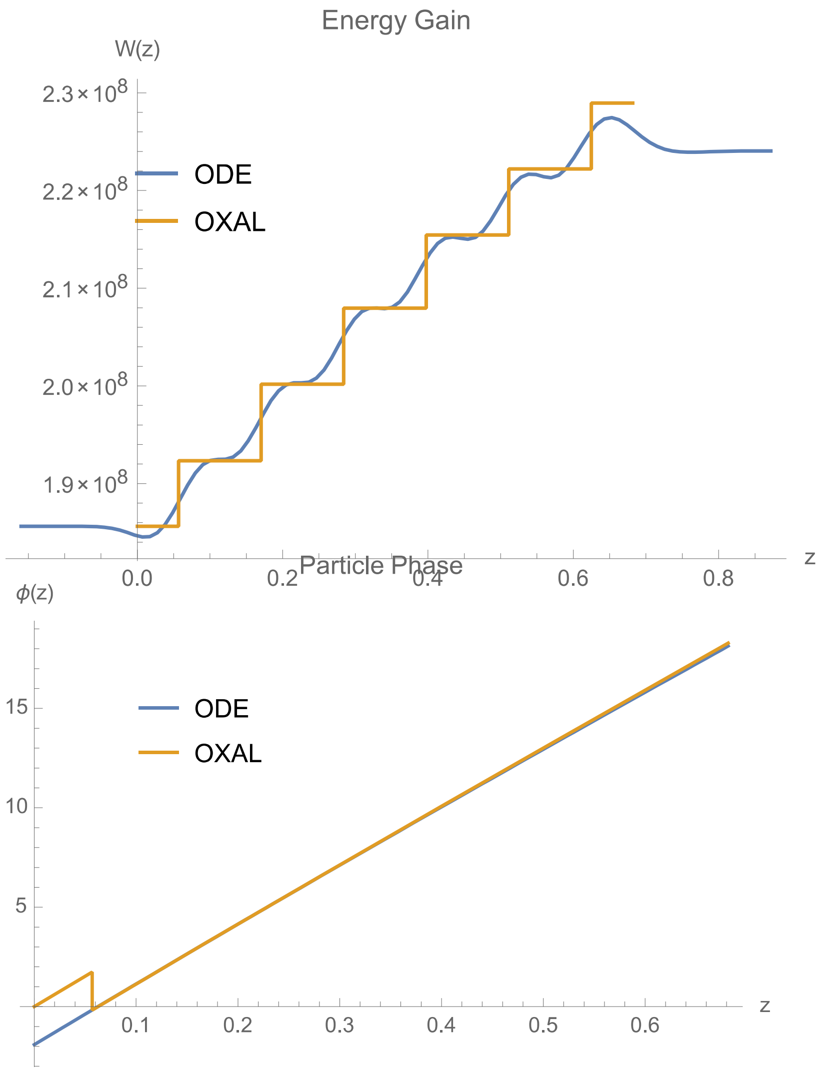




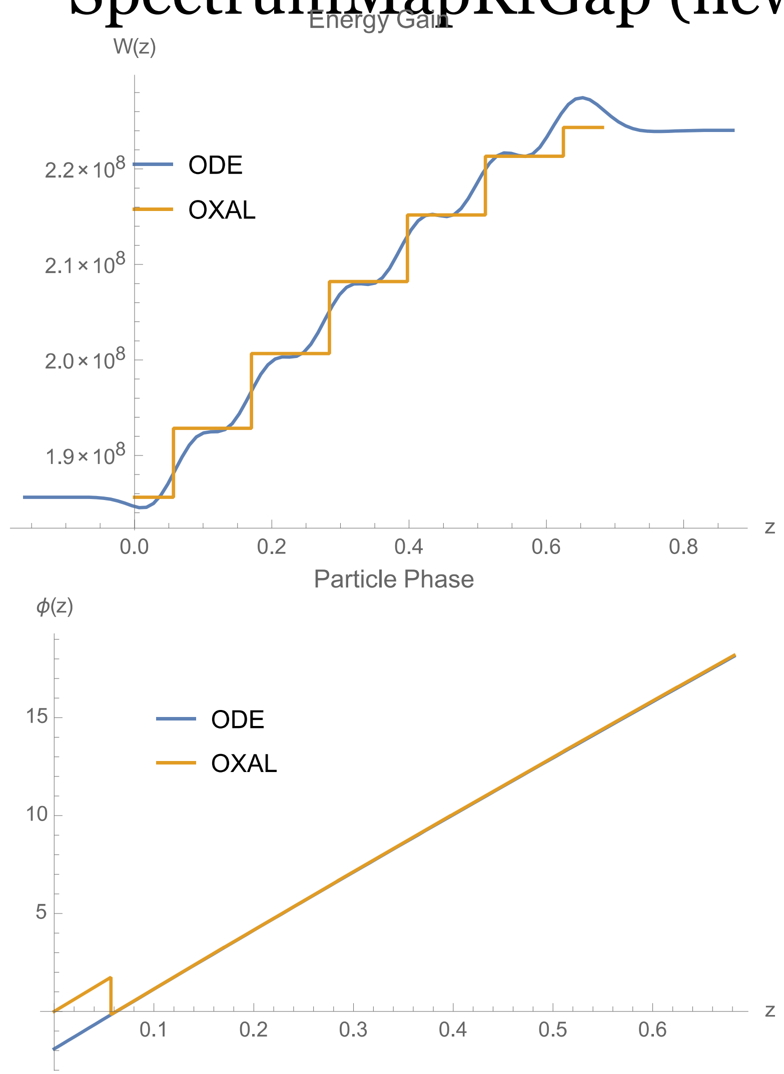
Cavity Simulation: SCL:Cav01a
Online Model vs. Analytic – Old and New

$W_m = 185.7\text{MeV}$
$\varphi_o = -10^o$

- IdealRfGap (old)



- SpectrumMapRfGap (new)



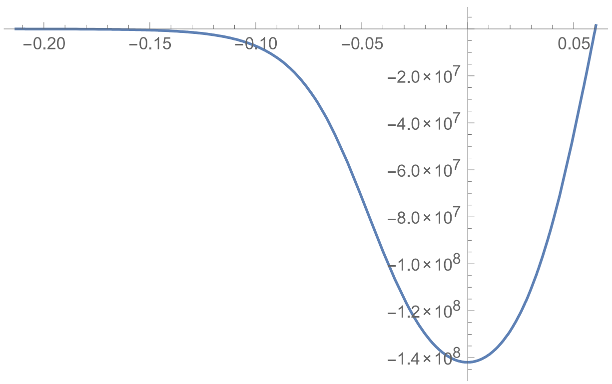
Modeling Issues

- Electrical center and length
  - from field map?
  - from geometry?
- Geometric offsets?
- Cavity drive and phase
  - Gap proportions
  - Gap phase w.r.t.?

Single-Gap: Hard-Edge Model

- Cav01a:Rg01Field Map

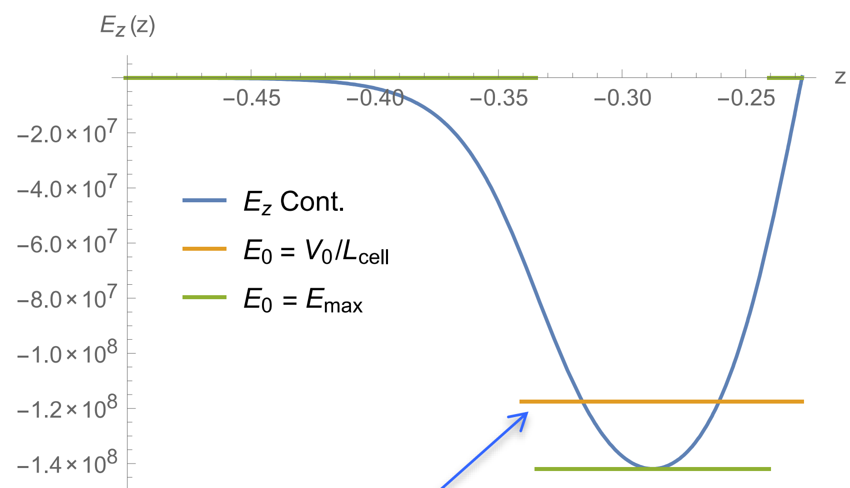
$$E_z(z) = E_z(r,z)\,|_{r=0}$$



- Hard-edge model (computer)

$$E_z(z) = \begin{cases} E_0 & \text{for } z \in [z_1, z_2] \\ 0 & \text{otherwise} \end{cases}$$

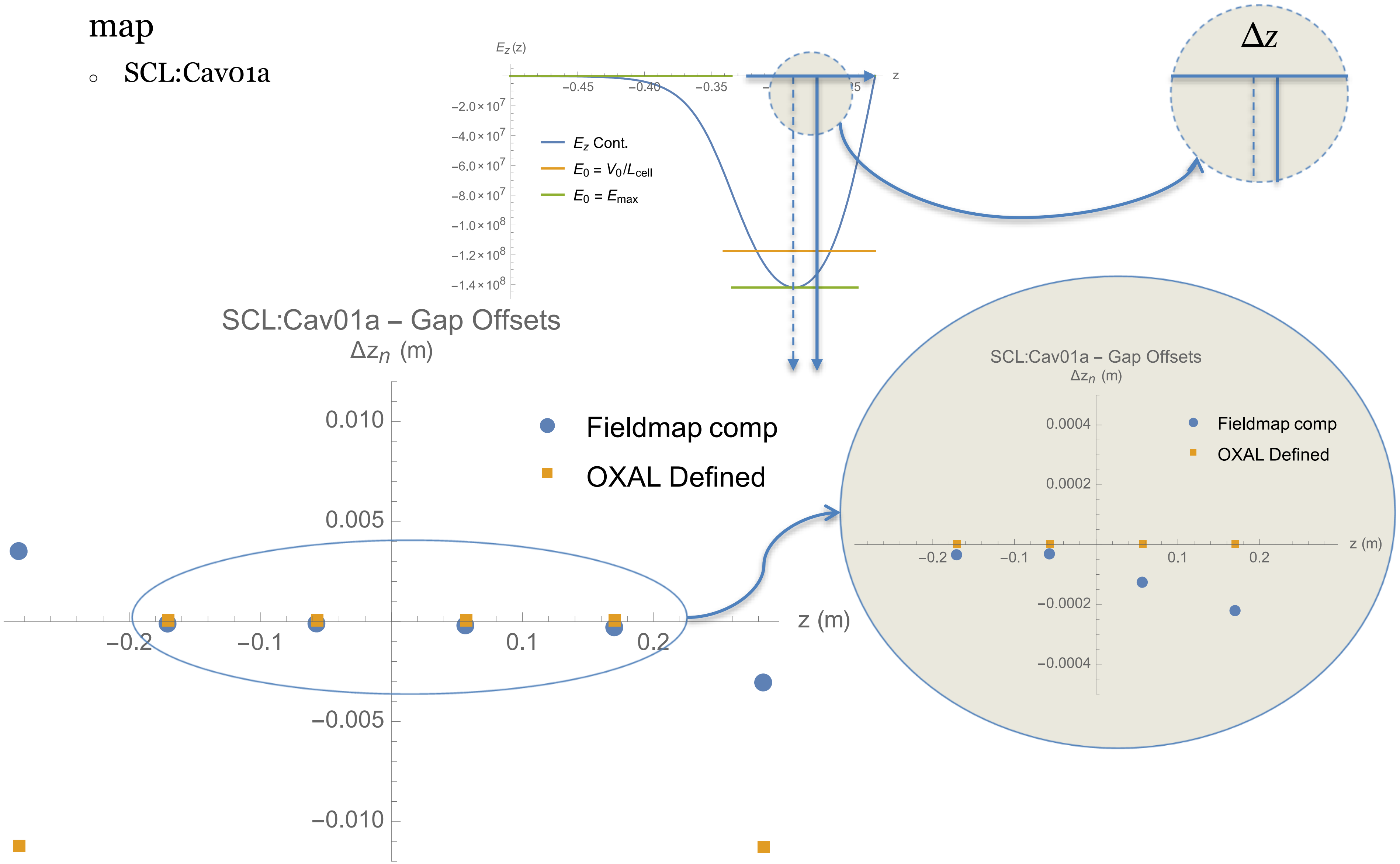
- <u>How do we pick E_o and length L_{eff}?</u>
  - E_o = avg field ($L_{eff} = L_{cell}$)
  - E_o = max field ($L_{eff} = V_o/E_{max}$)



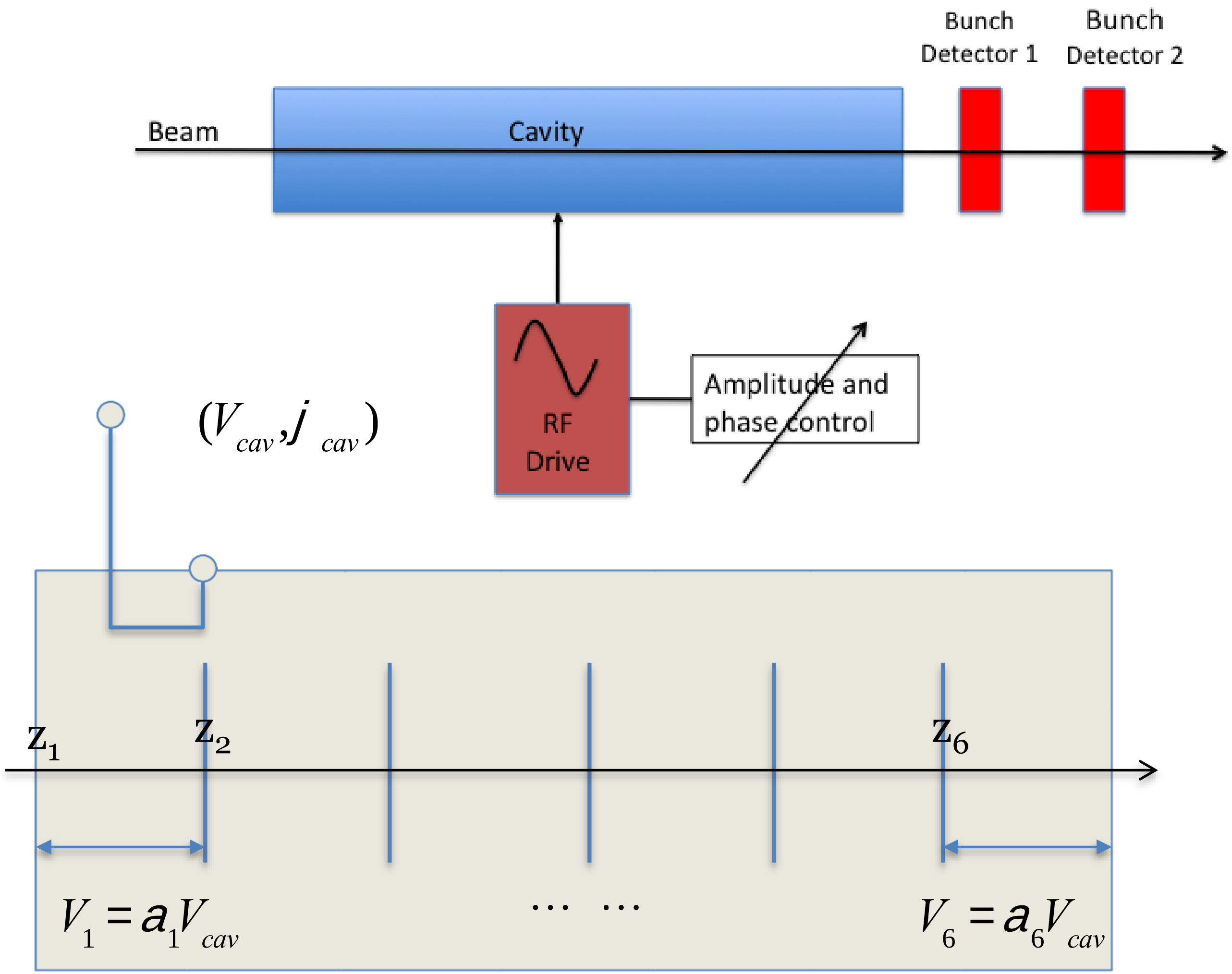
Geometry of cavity is default SNS database configuration

Modeling Issues: Gap Center Offsets

- Comparing value in SNS database (used in Open XAL) and those computed from field map
  - SCL:Cav01a



Modeling Issues: Driving the Cavity



Potential V_n across each gap is proportional to cavity voltage V_{cav}, say by α_n

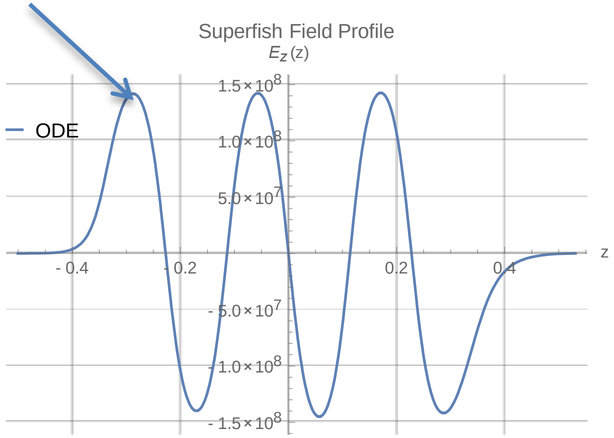
$$V_n = \int_{z_n}^{z_{n+1}} E_z(z)\,dz = \alpha_n V_{cav}$$

Modeling Issues: Driving the Cavity

Potential V_n across each gap proportional to cavity voltage V_{cav}

$$V_n = \int_{z_n}^{z_{n+1}} E_z(z)\,dz = \alpha_n V_{cav}$$

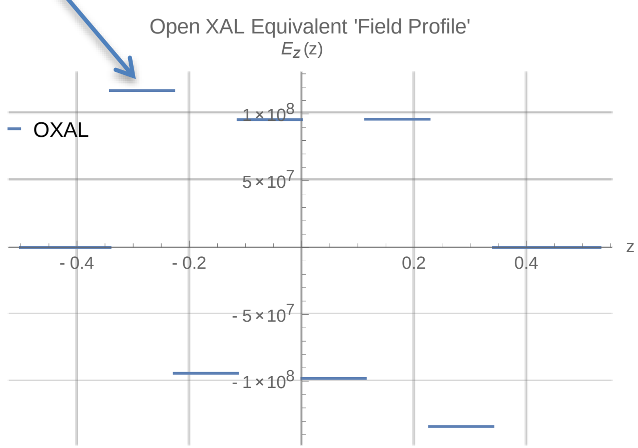
V_{cav}



Directly Computed

α_1	α_2	α_3	α_4	α_5	α_6
1.00	0.801	0.815	0.833	0.817	1.139

V_{cav}



SNS Database (XDXF File)

α_1	α_2	α_3	α_4	α_5	α_6
1.00	0.735	0.735	0.735	0.735	1.00

The drive proportions α_n used in Open XAL are different that those computed directly

Summary: Verification and Modeling Issues

- The upgrade appears to be computing what it is supposed to compute (verified)
  - There are still some bugs

- I need to resolve the modeling issues concerning gap offsets and gap drive proportions before I can request pull.

Full-Field, Thin-Lens RF Gap Model

Completing the Thin Lens Representation of RF Gaps with Arbitrary Axial Fields

(Theory Lite)

Story of the Open XAL RF Gap Model

Original motivation was to fix a necessary kluge in the RF gap, and also to provide dynamic phase tracking for beam probes

- The RF cavity effects and particle phases were contained in the RF gap element as “global dynamic variables” (original XAL Online Model did not support phase)

- Began a refactoring of the previous RF gap modeling element
  - Could not follow flow
  - Quantities were undefined, eg., $S' = dS/d\beta$ or dS/dk or $dS(\beta)/dk$ or $dS(k)/dk$, etc.

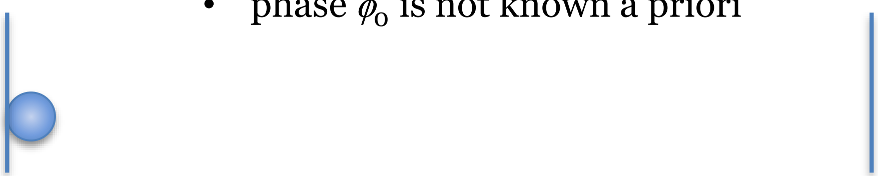
- I could not understand or follow the process
  - could not interpret the debugging output

- Retreated, then started again from first principles

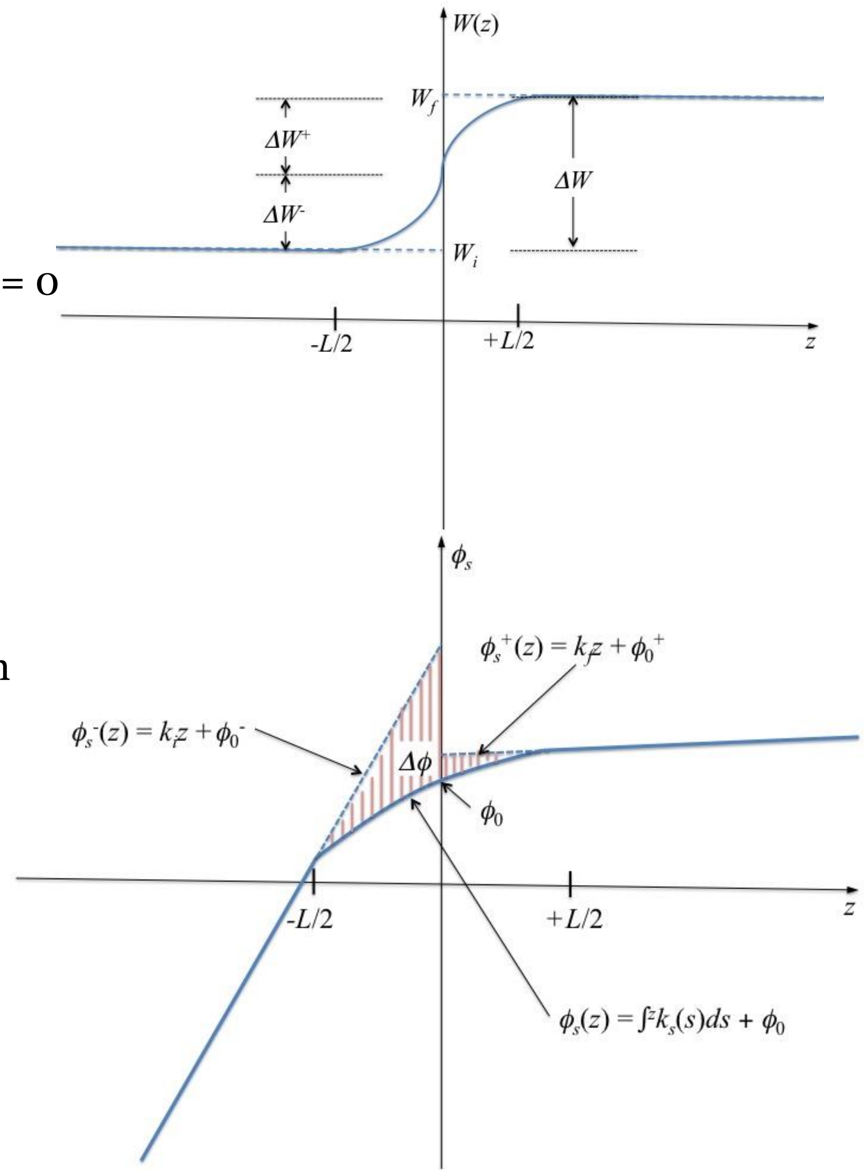
Start from the Very Very Beginning

Thin Lens Theory for Full-Field Gap

- *Full Field* means *S* is not zero
- Thin Lens model for RF gap:
  - particle coasts with wave number k_i to gap center $z = 0$
  - particle arrives at gap center with phase ϕ_0
    - phase ϕ_0 is not known a priori



  - particle experiences phase jump $\Delta\phi$ and energy gain ΔW
  - particle coasts with wave number k_f until next interaction region



This is everything! The rest is analysis.

Review: The Text Book RF Gap Equations

- Synchronous particle kinetic energy W_s through a longitudinal field E_z and resulting energy gain $\Delta W(k)$ from gap

$$W_s(z) = W_i + \int_{-\infty}^{z} qE_z(0,s)\cos[\omega t(s) + \phi_0]\, ds\,,$$

$$\approx W_i + q\cos\phi_0 \int_{-\infty}^{z} E_z(0,s)\cos\bar{k}s\, ds - q\sin\phi_0 \int_{-\infty}^{z} E_z(0,s)\sin\bar{k}s\, ds\;,$$

a special value for $k = (\omega/v)$

$$\Delta W(\bar{k}) \approx qV_0T(\bar{k})\cos\phi_0 - qV_0S(\bar{k})\sin\phi_0.$$

Fourier sine transform of E_z

Fourier cosine transform of E_z

Together "transit time factors" T and S produce the complete Fourier transform of E_z containing all its information

$$\mathcal{F}[E_z] = V_0T(k) - iV_0S(k)\,,$$
$$E_z(0,z) = \mathcal{F}^{-1}[V_0T - iV_0S]\,.$$

Review: Phase Jump

- Phase jump $\Delta\phi$ derived via a traveling "phase slip" $\delta\phi$ from the *synchronous phase*

$$\phi_s(z) = \int_0^z k_s(s)\,ds + \phi_0\,,$$

phase of the synchronous particle

$$\delta\phi_s(z) \approx -\frac{q}{2vW}\int_{-\infty}^{z} E_z(s;t)\,ds\,,$$

$$= \frac{qk}{2W}\int_{-\infty}^{+\infty}\int_0^{s} E_z(s;t)\,dsdz\,.$$

But wait!

- You cannot integrate through *z*=0.
- *k* must be held constant even though it's the variable we are computing.
- Line integrals are actually area integrals.

$$\Delta\phi_s = \int_{-\infty}^{+\infty} \delta\phi_s(z)dz$$

$$= \frac{qk}{2W}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_0^{s} E_z(s;t)\,dzdsdz\,,$$

$$= \frac{qV_0}{2W}k_0 T'(\bar{k})\sin\phi_0\,.$$

What is that?

Magic!

Encore – How to Compute the Right Answer

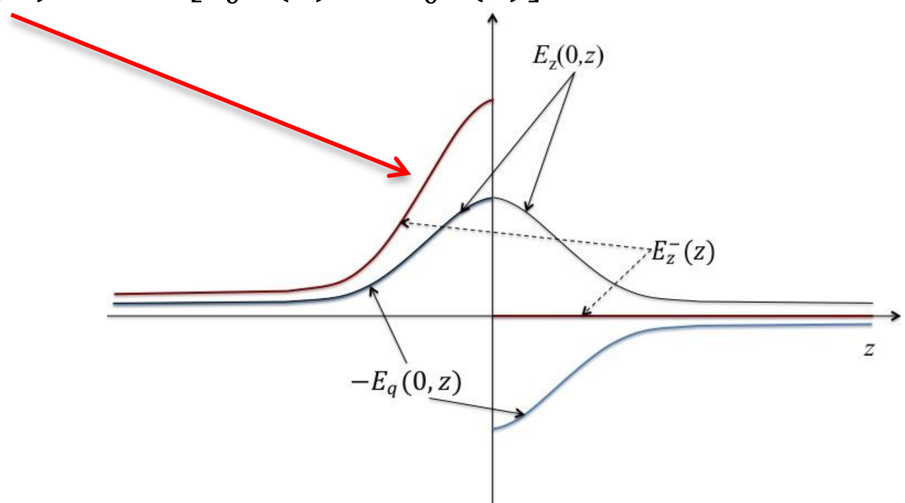
- If you want to compute the *right answer* …
  - Must compute special k at the "gap center " using the special formula

$$\Delta W(\bar{k}) \approx qV_0 T(\bar{k}) \cos\phi_0 - qV_0 S(\bar{k}) \sin\phi_0.$$

Well, S isn't really S, but a special S, one that is computed for the half field E_z^-, we just call it S

special S

$$E_z^-(0,z) = \mathcal{F}^{-1}[V_0 T(k) - iV_0 S(k)]$$



- This is completely justifiable because …
  - The real S is zero anyway because all fields are symmetric about z = 0.
  - **It gives you the right answer**

(It's important not to distinguish between S and S)

Derivations from Thin Lens Model w/ Full Fields

- Energy gain ΔW the same
  - with special S replaced by S
- Phase jump $\Delta\phi$ has two additional terms

ordinary S

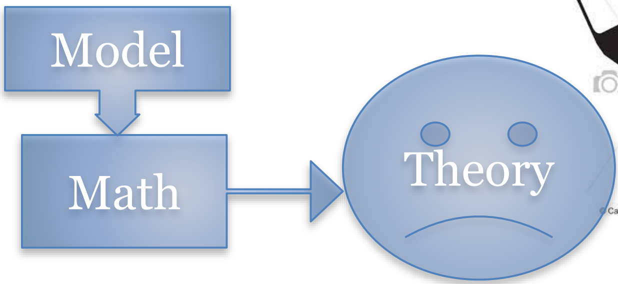
$$\Delta W(\bar{k}) \approx qV_0 T(\bar{k}) \cos\phi_0 - qV_0 S(\bar{k}) \sin\phi_0$$

$$\Delta\phi^- \approx -\frac{K_i}{2\pi}\cos\phi_0 \int_{-\infty}^{+\infty}\int_{-\infty}^{0}\left[sT_z(k)\cos ks\cos\bar{k}s + sS_z(k)\sin ks\cos\bar{k}s\right]ds\,dk\ ,$$

$$+\frac{K_i}{2\pi}\sin\phi_0 \int_{-\infty}^{+\infty}\int_{-\infty}^{0}\left[sT_z(k)\cos ks\sin\bar{k}s + sS_z(k)\sin ks\sin\bar{k}s\right]ds\,dk\ ,$$

brand new
(and resilient)

  - Could **prove** these terms not zero
    - Where do they come from?
  - I tried to reproduce the magic
    - Without success
  - Magic is stronger than math



Putting it together

$$\mathcal{E}_z(\sigma) \triangleq \mathcal{L}_2[E_z(z)](\sigma)$$ Laplace transform of field

$$\mathcal{E}_q(\sigma) = \frac{1}{\pi i}\int_{Br\varepsilon} \frac{1}{\sigma - s}\mathcal{E}_z(s)ds$$ "quadrature" field transform

- Laplace Transform Domain and Extra Terms
  - Extra terms come from a **quadrature** field $\mathcal{E}_q$ (Need a quadrature field E_q conj. to E_z)
  - Contains T_q and S_q conjugate to T_z and S_z

$$\mathcal{E}_q(\sigma) = \begin{cases} +2\mathcal{E}_z(\sigma) & \mathrm{Re}\,\sigma < 0\,, \\ -i\mathcal{H}[\mathcal{E}_z(\sigma)] & \mathrm{Re}\,\sigma = 0\,, \\ -2\mathcal{E}_z(\sigma) & \mathrm{Re}\,\sigma > 0, \end{cases}$$

we are stuck here

- Hilbert Transform
  - Quadrature T_q and S_q are Hilbert transforms of primary T_z and S_z
  - Spectral **pre-envelope** $\mathcal{E}^-$ quadrature sum of spectrum and conjugate spectrum
  - Inverse Laplace transform yields **magic field**

$$\mathcal{H}[f(t)] \triangleq PV\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{f(\tau)}{t-\tau}d\tau$$ Hilbert transform of function

$$\mathcal{E}_q(ik) = i\mathcal{H}[\mathcal{E}_z(ik)]\,, \quad = -\mathcal{H}[S_z(k) + iT_z(k)]\,.$$

$$\Rightarrow \quad T_q(k) = -\mathcal{H}[S_z(k)]\,, \quad S_q(k) = +\mathcal{H}[T_z(k)]\,.$$

- Hamiltonian
  - Pre-gap and post-gap Hamiltonians H^- and H^+ are pre-spectrum rotated by gap phase ϕ ("action-angle")
  - Dynamics – projection of Hamiltonian on imaginary axis

$$\mathcal{E}^-(ik) = \frac{\mathcal{E}_z(ik) - \mathcal{E}_q(ik)}{2}\,, \quad = \frac{T_z(k) - T_q(k)}{2} - i\,\frac{S_z(k) - S_q(k)}{2}$$

Special S field

$$E_z^-(z) = \begin{cases} E_z(0,z) & z < 0\,, \\ \frac{1}{2}E_z(0,z) & z = 0\,, \\ 0 & z > 0\,. \end{cases}$$

$$H^-(\phi,k) \triangleq \mathcal{E}^-(ik)e^{-i\phi}$$
$$H^+(\phi,k) \triangleq \mathcal{E}^+(ik)e^{-i\phi}$$

$$\Rightarrow$$

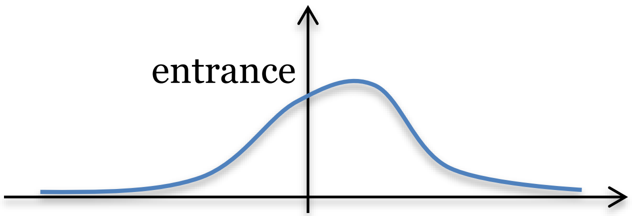
$$\Delta\phi^-(\phi,k) = +K_i\frac{d}{dk}\,\mathrm{Im}\,H^-(\phi,k)\,,$$

$$\Delta W^-(\phi,k) = -qV_0\frac{d}{d\phi}\,\mathrm{Im}\,H^-(\phi,k)\,.$$

Summary

- Analysis
  - Does it vindicate the magic?
  - Does it reveal how the trick is done?
  - Magic does not generalize

- Full field model
  - Require both TTFs T_z and S_z
    - Contains field asymmetries
    - Gap "offsets" are represented



- Thin-lens, full-field RF gap model
  - Requires primary TTFs T_z and S_z and derivatives
  - Requires quadrature TTFs T_q and S_q and derivatives
    - **Must** use "half gap calculation", **must** use half field, **must** use special $S = S_q$
    - In principle T_q and S_q can be computed from T_z and S_z via Hilbert transform,
  - Must compute phase jump and energy gain for **both** pre-gap and post-gap region

Transform Properties

- Hilbert Transform
  - Shifts phase of function by 90 degrees, amplitude unchanged – “quadrature”
  - Function and its Hilbert transform are orthogonal
  - Function and its Hilbert transform have same energy
- Pre-Envelope (Analytic Signals)
  - Have spectra of only one sign, + or –
  - (Complex) absolute value is signal envelope
  - Eg. $f(t) = \cos \omega t \;\Rightarrow f^{-}(t) = e^{+i\omega t}$

    $f(t) = \sin \omega t \;\Rightarrow f^{-}(t) = e^{-i\omega t}$

    $f(t) = m(t) \cos \omega t \;\Rightarrow f^{-}(t) = m(t) e^{+i\omega t}$

convenient for modulated signal

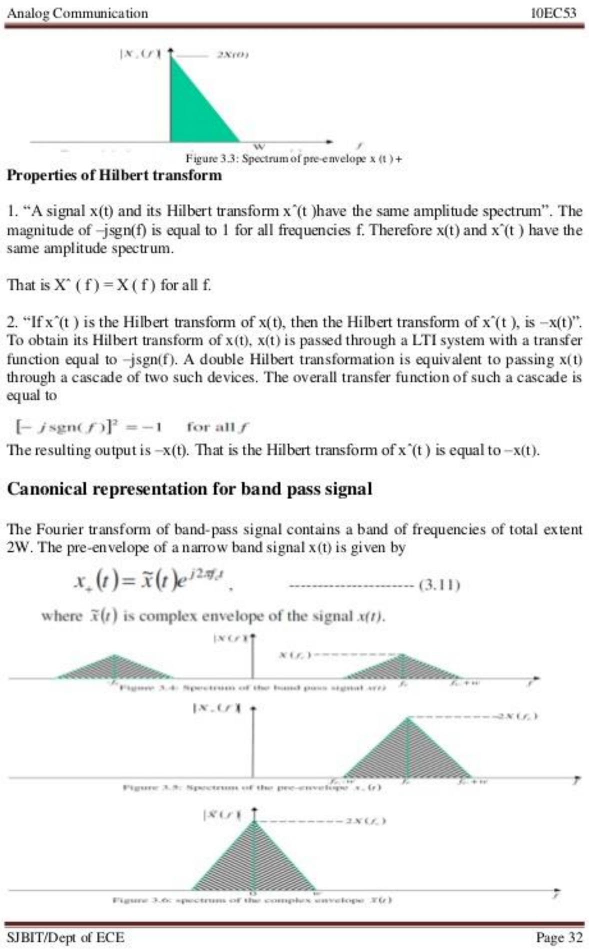

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Figure 3.3: Spectrum of pre-envelope x (t) +

Properties of Hilbert transform

1. “A signal x(t) and its Hilbert transform xˆ(t)have the same amplitude spectrum”. The magnitude of –jsgn(f) is equal to 1 for all frequencies f. Therefore x(t) and xˆ(t) have the same amplitude spectrum.

That is Xˆ (f) = X (f) for all f.

2. “If xˆ(t) is the Hilbert transform of x(t), then the Hilbert transform of xˆ(t), is –x(t)”. To obtain its Hilbert transform of x(t), x(t) is passed through a LTI system with a transfer function equal to –jsgn(f). A double Hilbert transformation is equivalent to passing x(t) through a cascade of two such devices. The overall transfer function of such a cascade is equal to

$$[-j\,\mathrm{sgn}(f)]^2 = -1 \quad \text{for all } f$$

The resulting output is –x(t). That is the Hilbert transform of xˆ(t) is equal to –x(t).

Canonical representation for band pass signal

The Fourier transform of band-pass signal contains a band of frequencies of total extent 2W. The pre-envelope of a narrow band signal x(t) is given by

$$x_+(t) = \tilde{x}(t) e^{j2\pi f_c t}, \quad \text{-----------------------} (3.11)$$

where $\tilde{x}(t)$ is complex envelope of the signal $x(t)$.

Figure 3.4: Spectrum of the band pass signal x(t)

Figure 3.5: Spectrum of the pre-envelope $x_+(t)$

Figure 3.6: spectrum of the complex envelope $\tilde{x}(t)$

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Dynamics Equations

- Pre-Envelope of a Function
  - Hilbert transform “completes” a function on the complex plane
  - TTF T needs special S to complete it
  - Full-field representation: Tz and Sz need Tq and Sq to complete them
- Hamiltonian
  - Spectral pre- and post-envelopes are boundaries of analytic functions on complex plane
  - Complex Hamiltonian equals that boundary rotated by ϕ_o degrees
  - Dynamics is projection of Hamiltonian onto the imaginary axis

No Magic

- Energy gain ΔW is equal to the potential in electric field
  - with spatial mode k^-
  - at time $\omega t = \phi_0$

$$\Delta W(\bar{k}) \approx qV_0 T(\bar{k}) \cos\phi_0 - qV_0 S(\bar{k}) \sin\phi_0.$$

maximum potential across gap