Open XAL Acceleration Model Upgrade
Architecture, Verification, Modeling Issues

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Objectives

• Create generalized (thin-lens) RF acceleration model
  o Better predict SNS linac (SCL) ?
  o No field restrictions

• Retire the currently used XAL version
  o Repair/replace earlier “quick fixes”
  o Hierarchical acceleration model.

full field = no assumptions
Tasks

• Needed RF cavity software model (hierarchical)
  o Need (major) modification to lattice generator
    to support nested modeling elements
  o Probes must carry their own phase information
  o Algorithms modified to support phase
  o RF cavity container/manager class
    • Cell indices, modes, amplitudes, phases, etc.
  o New RF gap class

generated a cascade of upgrades to Open XAL

• Need tools to compute new spectral quantities

• Architectural support for new data
  o storage, retrieval, encapsulation
Current Status

- All the aforementioned issues have been addressed.

- The new RF acceleration model has been analyzed and documented extensively.

- A tool for computing spectral quantities from fields maps has been written, *TTF Workshop*
  - C.K. Allen, James Ghawaly, with help from A. Shishlo

- Currently verifying and debugging the new acceleration model.
Software Architecture

RF Cavity Design

Figure 31: RF cavity software design

RF Gap Design

Handles all energy gain calculations for RF gap.

Replaces IdealRFCap
Spectral Data Locations

Superfish data

processed spectral data
Verification of Upgrades

• Currently verifying operation of acceleration model
  – particular relevance to the SCL
  – comparing results of online model to analytic model with direct numerical integration using Mathematica

verification – does the model compute what it is supposed to compute?

validation – does the model compute the answer? (reality)
**Verification:** Do they compute the same thing?

- **Computer Model**
  - Augmented Panofsky Eq.
  - Discrete, coupled, transcendental equations

\[
\begin{align*}
\phi_0 &= \phi_0^- + \text{Im} \left[ k - \frac{\partial}{\partial i k} H^- (\phi_0 + \kappa^-) \right], \\
W_0 &= W^- - \text{Im} \left[ \frac{\partial}{\partial \phi} H^- (\phi_0 + \kappa^-) \right], \\
\end{align*}
\]

where

\[
H^- (\phi + ik) \triangleq qV_0 e^{-i\phi} \varepsilon^-(ik), \\
H^+ (\phi + ik) \triangleq qV_0 e^{-i\phi} \varepsilon^+(ik).
\]

- **Analytic Model**
  - Coupled 1\textsuperscript{st} order ODEs

\[
\begin{align*}
\phi'(z) &= k[W(z)], \\
W'(z) &= qE_z(z) \cos \phi(z),
\end{align*}
\]

Longitudinal electric field (defines the gap)

Laplace transform spectrum of \( E_z(z) \) (defines the gap)
Single-Gap:
Test Example Medium-β SCL:Cav01a:Rg01

- Single gap: SCL:Cav01a:Rg01
- Superfish Field Map: SCL:Cav01a:Rg01- Rg06
Single-Gap: Simulation SCL:Cav01a:Rg01
Online Model vs. Analytic – Old and New

- **IdealRfGap (old)**
  - $W(z)$
  - $\phi(z)$

- **SpectrumMapRfGap (new)**
  - $W(z)$
  - $\phi(z)$
Cavity Analysis

\[ E_0 = \frac{V_0}{L_{\text{cell}}} \]

\[ E_0 = E_{\text{max}} - 0.4 - 0.2 + 0.2 + 0.4 \]

\[ E_z(z) \]

- \( E_z \) Cont.
- \( E_0 = \frac{V_0}{L_{\text{cell}}} \)
- \( E_0 = E_{\text{max}} \)
Cavity Simulation: SCL:Cav01a

Online Model vs. Analytic – Old and New

- **IdealRfGap (old)**
  - $W_m = 185.7\text{MeV}$
  - $\phi_0 = -10^\circ$

- **SpectrumMapRfGap (new)**
Modeling Issues

- Electrical center and length
  - from field map?
  - from geometry?
- Geometric offsets?
- Cavity drive and phase
  - Gap proportions
  - Gap phase w.r.t.?
Single-Gap: Hard-Edge Model

- Cav01a: Rg01 Field Map

\[ E_z(z) = E_z(r,z) \bigg|_{r=0} \]

- Hard-edge model (computer)

\[ E_z(z) = \begin{cases} E_0 & \text{for } z \in [z_1, z_2] \\ 0 & \text{otherwise} \end{cases} \]

- How do we pick \( E_0 \) and length \( L_{\text{eff}} \)?
  - \( E_0 = \text{avg field} \) (\( L_{\text{eff}} = L_{\text{cell}} \))
  - \( E_0 = \text{max field} \) (\( L_{\text{eff}} = V_0/E_{\text{max}} \))

Geometry of cavity is default SNS database configuration
Modeling Issues: Gap Center Offsets

- Comparing value in SNS database (used in Open XAL) and those computed from field map
  - SCL:Cav01a
Modeling Issues: Driving the Cavity

Potential $V_n$ across each gap is proportional to cavity voltage $V_{cav}$, say by $\alpha_n$

$$V_n = \int_{z_n}^{z_{n+1}} E_z(z)\,dz = \alpha_n V_{cav}$$

$$V_1 = V_{cav} \quad \cdots \quad V_6 = 6 V_{cav}$$
Modeling Issues: Driving the Cavity

Potential $V_n$ across each gap proportional to cavity voltage $V_{cav}$

$$V_n = \int_{z_n}^{z_{n+1}} E_z(z) \, dz = n \cdot V_{cav}$$

**Superfish Field Profile**

<table>
<thead>
<tr>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\alpha_4$</th>
<th>$\alpha_5$</th>
<th>$\alpha_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.801</td>
<td>0.815</td>
<td>0.833</td>
<td>0.817</td>
<td>1.139</td>
</tr>
</tbody>
</table>

**Open XAL Equivalent ‘Field Profile’**

<table>
<thead>
<tr>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\alpha_4$</th>
<th>$\alpha_5$</th>
<th>$\alpha_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.735</td>
<td>0.735</td>
<td>0.735</td>
<td>0.735</td>
<td>1.00</td>
</tr>
</tbody>
</table>

The drive proportions $\alpha_n$ used in Open XAL are different than those computed directly.

The drive proportions $\alpha_n$ used in Open XAL are different than those computed directly.
Summary: Verification and Modeling Issues

• The upgrade appears to be computing what it is supposed to compute (verified)
  ○ There are still some bugs

• I need to resolve the modeling issues concerning gap offsets and gap drive proportions before I can request pull.
Full-Field, Thin-Lens RF Gap Model

Completing the Thin Lens Representation of RF Gaps with Arbitrary Axial Fields

(Theory Lite)
Story of the Open XAL RF Gap Model

Original motivation was to fix a necessary kluge in the RF gap, and also to provide dynamic phase tracking for beam probes

- The RF cavity effects and particle phases were contained in the RF gap element as “global dynamic variables” (original XAL Online Model did not support phase)

- Began a refactoring of the previous RF gap modeling element
  - Could not follow flow
  - Quantities were undefined, eg., $S' = dS/d\beta$ or $dS/dk$ or $dS(\beta)/dk$ or $dS(k)/dk$, etc.

- I could not understand or follow the process
  - could not interpret the debugging output

- Retreated, then started again from first principles
Start from the Very Very Beginning

Thin Lens Theory for Full-Field Gap

- **Full Field** means $S$ is not zero
- Thin Lens model for RF gap:
  - particle coasts with wave number $k_i$ to gap center $z = 0$
  - particle arrives at gap center with phase $\phi_0$
    - phase $\phi_0$ is not known a priori
  - particle experiences phase jump $\Delta\phi$ and energy gain $\Delta W$
  - particle coasts with wave number $k_f$ until next interaction region

This is everything! The rest is analysis.
Review: The Text Book RF Gap Equations

- Synchronous particle kinetic energy $W_s$ through a longitudinal field $E_z$ and resulting energy gain $\Delta W(k)$ from gap

\[
W_s(z) = W_i + \int_{-\infty}^{z} qE_z(0,s) \cos[\omega t(s) + \phi_0] \, ds ,
\]

\[
\approx W_i + q \cos \phi_0 \int_{-\infty}^{z} E_z(0,s) \cos k s \, ds - q \sin \phi_0 \int_{-\infty}^{z} E_z(0,s) \sin k s \, ds ,
\]

a special value for $k = (\omega/v)$

\[
\Delta W(k) \approx qV_0T(k) \cos \phi_0 - qV_0S(k) \sin \phi_0 .
\]

Together “transit time factors” $T$ and $S$ produce the complete Fourier transform of $E_z$ containing all its information

\[
\mathcal{F}[E_z] = V_0T(k) - iV_0S(k) ,
\]

\[
E_z(0,z) = \mathcal{F}^{-1}[V_0T - iV_0S] .
\]
Review: Phase Jump

- Phase jump $\Delta \phi$ derived via a traveling “phase slip” $\delta \phi$ from the synchronous phase

$$\phi_s(z) = \int_0^z k_s(s) \, ds + \phi_0,$$

phase of the synchronous particle

$$\delta \phi_s(z) \approx -\frac{q}{2\nu W} \int_0^z E_z(s; t) \, ds,$$

$$= \frac{qk}{2W} \int_{-\infty}^{+\infty} \int_{-\infty}^{0} \int_{0}^{+\infty} E_z(s; t) \, dsdz.$$

But wait!
- You cannot integrate through $z=0$.
- $k$ must be held constant even though it’s the variable we are computing.
- Line integrals are actually area integrals.

$$\Delta \phi_s = \int_{-\infty}^{+\infty} \delta \phi_s(z) \, dz$$

$$= \frac{qk}{2W} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{0} E_z(s; t) \, dzdsdz,$$

$$= \frac{qV_0}{2W} k_0 T'(\bar{k}) \sin \phi_0.$$
Encore – How to Compute the Right Answer

• If you want to compute the right answer ...
  o Must compute special $k$ at the “gap center“ using the special formula

\[ \Delta W(\bar{k}) \approx qV_0 T(\bar{k}) \cos \phi_0 - qV_0 S(\bar{k}) \sin \phi_0. \]

Well, $S$ isn’t really $S$, but a special $S$, one that is computed for the half field $E_z^{-}$, we just call it $S$

\[ E_z^-(0, z) = \mathcal{F}^{-1}[V_0 T(k) - iV_0 S(k)] \]

• This is completely justifiable because ...
  o The real $S$ is zero anyway because all fields are symmetric about $z = 0$.
  o It gives you the right answer

(It’s important not to distinguish between $S$ and $S$)
Derivations from Thin Lens Model w/ Full Fields

- Energy gain $\Delta W$ the same
  - with special $S$ replaced by $S$
  $$\Delta W(\bar{k}) \approx qV_0 T(\bar{k}) \cos \phi_0 - qV_0 S(\bar{k}) \sin \phi_0$$
- Phase jump $\Delta \phi$ has two additional terms
  - Could prove these terms not zero
    - Where do they come from?
  - I tried to reproduce the magic
    - Without success
  - Magic is stronger than math
Putting it together

- **Laplace Transform Domain and Extra Terms**
  - Extra terms come from a **quadrature** field \( \mathcal{E}_q \)
    (Need a quadrature field \( E_q \) conj. to \( E_z \))
  - Contains \( T_q \) and \( S_q \) conjugate to \( T_z \) and \( S_z \)
- **Hilbert Transform**
  - Quadrature \( T_q \) and \( S_q \) are Hilbert transforms of primary \( T_z \) and \( S_z \)
  - Spectral **pre-envelope** \( \mathcal{E} \) quadrature sum of spectrum and conjugate spectrum
  - Inverse Laplace transform yields **magic field**
- **Hamiltonian**
  - Pre-gap and post-gap Hamiltonians \( H^- \) and \( H^+ \) are pre-spectrum rotated by gap phase \( \phi \) ("action-angle")
  - Dynamics – projection of Hamiltonian on imaginary axis

\[
\mathcal{E}_z(\sigma) \triangleq \mathcal{L}_z[E_z(z)](\sigma) \quad \text{Laplace transform of field}
\]

\[
\mathcal{E}_q(\sigma) = \frac{1}{\pi i} \int_{Br\mathcal{E}} \frac{1}{\sigma - s} \mathcal{E}_z(s) ds \quad \text{“quadrature” field transform}
\]

\[
\mathcal{E}_q(\sigma) = \begin{cases} 
+2\mathcal{E}_z(\sigma) & \text{Re } \sigma < 0, \\
-i\mathcal{H}[\mathcal{E}_z(\sigma)] & \text{Re } \sigma = 0, \\
-2\mathcal{E}_z(\sigma) & \text{Re } \sigma > 0,
\end{cases}
\]

\[
\mathcal{H}[f(t)] \triangleq \text{PV} \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{f(\tau)}{t - \tau} d\tau \quad \text{Hilbert transform of function}
\]

\[
\mathcal{E}_q(ik) = i\mathcal{H}[\mathcal{E}_z(ik)], \quad T_q(k) = -\mathcal{H}[S_z(k)], \quad S_q(k) = +\mathcal{H}[T_z(k)].
\]

\[
\mathcal{E}^-_z(k) = \frac{\mathcal{E}_z(ik) - \mathcal{E}_q(ik)}{2},
\]

\[
= \frac{T_z(k) - T_q(k)}{2} - i \frac{S_z(k) - S_q(k)}{2}
\]

\[
E_z^-(z) = \begin{cases} 
\frac{1}{2} E_z(0,z) & z < 0, \\
\frac{1}{2} E_z(0,z) & z = 0, \\
0 & z > 0.
\end{cases}
\]

\[
\Delta \phi^-(\phi, k) = +K_i \frac{d}{dk} \text{Im } H^-(\phi, k),
\]

\[
\Delta W^-(\phi, k) = -qV_0 \frac{d}{d\phi} \text{Im } H^-(\phi, k).
\]
Summary

• Analysis
  o Does it vindicate the magic?
  o Does it reveal how the trick is done?
  o Magic does not generalize

• Full field model
  o Require both TTFs $T_z$ and $S_z$
    • Contains field asymmetries
    • Gap “offsets” are represented

• Thin-lens, full-field RF gap model
  o Requires primary TTFs $T_z$ and $S_z$ and derivatives
  o Requires quadrature TTFs $T_q$ and $S_q$ and derivatives
    • Must use “half gap calculation”, must use half field, must use special $S = S_q$
    • In principle $T_q$ and $S_q$ can be computed from $T_z$ and $S_z$ via Hilbert transform,
  o Must compute phase jump and energy gain for both pre-gap and post-gap region
Transform Properties

- **Hilbert Transform**
  - Shifts phase of function by 90 degrees, amplitude unchanged – “quadrature”
  - Function and its Hilbert transform are orthogonal
  - Function and its Hilbert transform have same energy

- **Pre-Envelope (Analytic Signals)**
  - Have spectra of only one sign, + or –
  - (Complex) absolute value is signal envelope
  - Eg. $f(t) = \cos \omega t \Rightarrow f^-(t) = e^{+i\omega t}$
    
    $f(t) = \sin \omega t \Rightarrow f^-(t) = e^{-i\omega t}$
    
    $f(t) = m(t) \cos \omega t \Rightarrow f^-(t) = m(t)e^{+i\omega t}$

Properties of Hilbert transform

1. “A signal $x(t)$ and its Hilbert transform $x'(t)$ have the same amplitude spectrum. The magnitude of $\text{sign}(f)$ is equal to 1 for all frequencies $f$. Therefore $x(t)$ and $x'(t)$ have the same amplitude spectrum.

   That is $X'(f) = X(f)$ for all $f$.

2. “If $x(t)$ is the Hilbert transform of $x(t)$, then the Hilbert transform of $x'(t)$, is $-x(t)$”. To obtain its Hilbert transform of $x(t)$, $x(t)$ is passed through a LTI system with a transfer function equal to $i\text{sign}(f)$. A double Hilbert transformation is equivalent to passing $x(t)$ through a cascade of two such devices. The overall transfer function of such a cascade is equal to $[-\text{sign}(f)]^2 = -1$ for all $f$.

   The resulting output is $-x(t)$. That is the Hilbert transform of $x'(t)$ is equal to $-x(t)$.

Canonical representation for band pass signal

The Fourier transform of band-pass signal contains a band of frequencies of total extent $2\omega$. The pre-envelope of a narrow band signal $x(t)$ is given by

$$x_c(t) = \tilde{x}(t) e^{+j2\omega t}, \quad (3.11)$$

where $\tilde{x}(t)$ is complex envelope of the signal $x(t)$.
Dynamics Equations

• Pre-Envelope of a Function
  o Hilbert transform “completes” a function on the complex plane
  o TTF $T$ needs special $S$ to complete it
  o Full-field representation: $T_z$ and $S_z$ need $T_q$ and $S_q$ to complete them

• Hamiltonian
  o Spectral pre- and post-envelopes are boundaries of analytic functions on complex plane
  o Complex Hamiltonian equals that boundary rotated by $\phi_0$ degrees
  o Dynamics is projection of Hamiltonian onto the imaginary axis
No Magic

- Energy gain $\Delta W$ is equal to the potential in electric field
  - with spatial mode $k$
  - at time $\omega t = \phi_0$

$$\Delta W(\bar{k}) \approx qV_0 T(\bar{k}) \cos \phi_0 - qV_0 S(\bar{k}) \sin \phi_0.$$

maximum potential across gap